ELEMENTARY

TEXT-BOOK

OF

PHYSICS.

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ILLUSTRATED BY NUMEROUS ENGRAVINGS ON WOOD.

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PREFACE.

THIS book is primarily intended as a text-book for elementary classes of Physics. It aims at presenting, in brief space, those portions of Theoretical Physics which are most essential as a foundation for subsequent advances, while at the same time most fitted for exercising the learner in logical and consecutive thought. It does not give minute directions for manipulation; but, avoiding details as much as possible, presents a connected outline of the main points of theory.

In order to place science upon an equal footing with the more established studies of ancient languages and mathematics, as a means of practically training the bulk of our youth to vigorous thought, it seems necessary that science text-books should be constructed upon such lines as these. It is not practicable to make the bulk of the boys in our public schools expert scientific manipulators; but it is practicable to ground them well in the main lines of scientific theory. The aim must be not so much to teach them many facts, as to teach them rightly to connect a few great facts together. Science must be taught them from a liberal, not from a technical stand-point.

The book is not intended to supersede oral instruction, but rather to create a demand for amplification and illustration such as the teacher will supply.

PREFACE TO THE SECOND EDITION.

In this edition the section on Dynamics has been re-written and made more elementary; a few of the more difficult articles have also been struck out from some of the other sections. British measures have been more largely employed, and everything relating to "absolute units" has been omitted. The book has thus been made easier for beginners, and better adapted to serve as an introductory text-book to the later editions of our "Deschanel," where the subjects now omitted are fully discussed.

Belfast, 1883.

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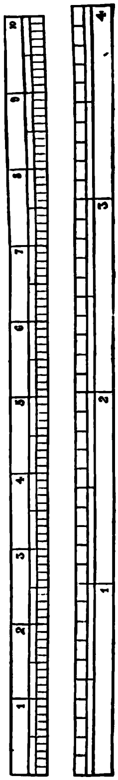


INCHES AND TENTHS.

1 millimetre = ·03937 inch, or about $\frac{1}{25}$ inch.
1 centimetre = ·3937 inch.
1 decimetre = 3·937 inches.
1 metre = 39·37 inches = 3·281 feet.

1 sq. centimetre = ·155 sq. in.

1 cub. centimetre = ·061025 cub. in.

1 gramme = 15·432 grain.
1 kilogramme = 2·205 lbs. avoir.

1 gramme per sq. centim. = 2·0481 lbs. per sq. ft.

1 kilogrammetre = 7·2331 foot-pounds.

1 inch = 2·54 centimetres.
1 foot = 30·48 centimetres.

1 sq. inch = 6·45 sq. centimetres.
1 sq. foot = 929 sq. centimetres.

1 cub. inch = 16·39 cub. centimetres.
1 cub. foot = 28316 cub. centimetres.

1 grain = ·0648 gramme.
1 oz. avoir. = 28·35 grammes.
1 lb. avoir. = 453·6 grammes.

TABLE OF RELATIVE DENSITIES.

LIQUIDS.

Pure water at 4° C.,	1·000
Sea water, ordinary,	1·026
Alcohol, pure,	·791
" proof spirit,	·916
Ether,	·716
Mercury at 0° C.,	13·596

SOLIDS.

Brass,	7·8 to 8·6
Copper,	8·6 to 8·9
Gold,	19 to 19·6
Iron, cast,	6·9 to 7·3
" wrought,	7·6 to 7·8
Lead,	11·4
Platinum,	21 to 22
Silver,	10·5
Steel,	7·8 to 7·9
Tin,	7·3 to 7·5
Zinc,	6·8 to 7·2
Ice,	·92
Basalt,	3·0
Brick,	2·0 to 2·2
Brickwork,	1·8
Chalk,	1·8 to 2·8
Clay,	1·9
Glass, crown,	2·5
" flint,	3·0
Quartz (rock crystal),	2·65
Sand,	1·4
Fir, spruce,	·48 to ·7
Oak, European,	·69 to ·99
Lignum vitæ,	·65 to 1·33
Sulphur, octahedral,	2·05
" prismatic,	1·98

GASES, at 0° C., and a pressure of 76 centimetres of mercury (one atmosphere).

Air, dry,	·0012932
Oxygen,	·0014298
Nitrogen,	·0012562
Hydrogen,	·00008957
Carbonic acid,	·0019774

ELEMENTARY PHYSICS.

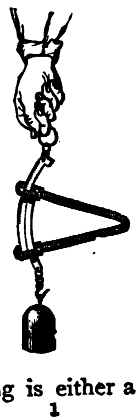
DYNAMICS.

FORCES AND THEIR COMPOSITION.

1. The science which treats of the laws of force is called by the most eminent modern authorities *dynamics* (from δύναμις, force). It is very frequently called *mechanics* (from μηχανή, a machine).

2. Force may be defined as that which tends to produce motion in a body at rest, or to produce change of motion in a body which is moving. A body is said to have uniform or unchanged motion when it moves in a straight line with uniform velocity; and every deviation of bodies from such motion is due to forces acting on them.

Fig. 1.



3. **Instruments for Measuring Force.**—We obtain the idea of force through our own conscious exercise of muscular force, and we can approximately estimate the amount of a force (if not too great or too small) by the effort which we have to make to resist it; as when we try the weight of a body by lifting it.

Spring-balances are instruments in which force is measured by means of its effect in bending or otherwise distorting elastic springs. One of the simplest forms is shown in Fig. 1. Very frequently the spring is either a

flat spiral (like the mainspring of a watch), or a helix (resembling a cork-screw).

A force may also be measured by causing it to act vertically downwards upon one of the scale-pans of a balance and counterpoising it by weights in the other pan.

4. Gravitation Units of Force.—In whatever way the measurement of a force is effected, the result, that is, the magnitude of the force, is usually stated in terms of weight; for example, in pounds or in kilogrammes. Such units of force (called gravitation units) are to a certain extent indefinite, inasmuch as gravity is not exactly the same over the whole surface of the earth; but they are sufficiently definite for ordinary commercial purposes.

5. Equilibrium, Statics, Kinetics.—When a body free to move is acted on by forces which do not move it, these forces are said to be *in equilibrium*, or to *equilibrate* each other. They may equally well be described as *balancing* each other. Dynamics is usually divided into two branches. The first branch, called *Statics*, treats of the conditions of equilibrium. The second branch, called *Kinetics*, treats of the movements produced by forces not in equilibrium.

6. Action and Reaction.—Experiment shows that force is always a mutual action between two portions of matter. When a body is urged by a force, this force is exerted by some other body, which is itself urged in the opposite direction with an equal force. When I press the table downwards with my hand, the table presses my hand upwards; when a weight hangs by a cord attached to a beam, the cord serves to transmit force between the beam and the weight, so that, by the instrumentality of the cord, the beam pulls the weight upwards and the weight pulls the beam downwards. Electricity and magnetism furnish no exception to this universal law. When a magnet attracts a piece of iron, the piece of iron attracts the magnet with a precisely equal force.

7. Specification of a Force acting at a Point.—Force may be applied over a finite area, as when I press

the table with my hand; or may be applied through the whole substance of a body, as in the case of gravity; but it is usual to begin by discussing the action of forces applied to a *single particle*, in which case each force is supposed to act along a mathematical straight line, and the particle or material point to which it is applied is called its *point* of application. A force is completely specified when its *magnitude*, its *point of application*, and its *line of action* are all given.

8. Conditions of Equilibrium for Two Forces.—In order that two forces applied to a rigid body should be in equilibrium, it is necessary and sufficient that they fulfil the following conditions:—

1st. Their lines of action must be one and the same.

2nd. The forces must act in opposite directions along this common line.

3rd. They must be equal in magnitude.

It will be observed that nothing is said here about the points of application of the forces. They may in fact be anywhere upon the common line of action. *The effect of a force upon a rigid body is not altered by changing its point of application to any other point in its line of action.* This is called the principle of the *transmissibility of force.*

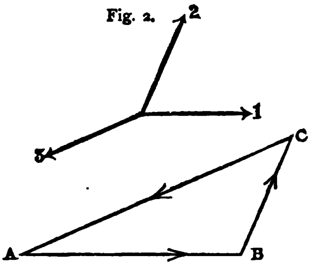
It follows from this principle that the condition of equilibrium for any number of forces with the same line of action is simply that the sum of those which act in one direction shall be equal to the sum of those which act in the opposite direction.

9. Three Forces meeting in a Point. Triangle of Forces.—If three forces, not having one and the same line of action, are in equilibrium, their lines of action must lie in one plane, and must either meet in a point or be parallel. We shall first discuss the case in which they meet in a point.

From any point A (Fig. 2) draw a line A B parallel to one of the two given forces, and so that in travelling from A to B we should be travelling in the same direction in which the force acts (not in the opposite direction). Also

let it be understood that the length of A B represents the magnitude of the force.

Fig. 2.



From the point B draw a line B C representing the second force in direction, and on the same scale of magnitude on which A B represents the first.

Then the line C A will represent both in direction and magnitude the third force which would equilibrate the first two.

The principle embodied in this construction is called the *triangle of forces.* It may be briefly stated as follows: —*The condition of equilibrium for three forces acting at a point is, that they be represented in magnitude and direction by the three sides of a triangle, taken one way round.* The meaning of the words "taken one way round" will be understood from an inspection of the arrows with which the sides of the triangle in Fig. 2 are marked. If the directions of all three arrows are reversed the forces represented will still be in equilibrium. The arrows must be so directed that it would be possible to travel completely round the triangle by moving along the sides in the directions indicated.

When a line is used to represent a force, it is always necessary to employ an arrow or some other mark of direction, in order to avoid ambiguity between the direction intended and its opposite. In naming such a line by means of two letters, one at each end of it, the order of the letters should indicate the direction intended. The direction of A B is from A to B; the direction of B A is from B to A.

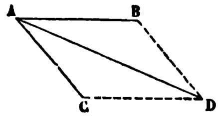
10. Resultant and Components.—Since two forces acting at a point can be balanced by a single force, it is obvious that they are equivalent to a single force, namely, to a force equal and opposite to that which would

balance them. This force to which they are equivalent is called their *resultant*. Whenever one force acting on a rigid body is equivalent to two or more forces, it is called their resultant, and they are called its *components*. When any number of forces are in equilibrium, a force equal and opposite to any one of them is the resultant of all the rest.

The "triangle of forces" gives us the resultant of any two forces acting at a point. For example, in Fig. 2, A C (with the arrow in the figure reversed) represents the resultant of the forces represented by A B and B C.

11. Parallelogram of Forces.—The proposition called the "parallelogram of forces" is not essentially distinct from the "triangle of forces," but merely expresses the same fact from a slightly different point of view. It is as follows:—*If two forces acting upon the same rigid body in lines which meet in a point, be represented by two lines drawn from the point, and a parallelogram be constructed on these lines, the diagonal drawn from this point to the opposite corner of the parallelogram represents the resultant.* For example, if A B, A C, Fig. 3, represent the two forces, A D will represent their resultant.

Fig. 3.



To show the identity of this proposition with the triangle of forces, we have only to substitute B D for A C (which is equal and parallel to it). We have then two forces represented by A B, B D (two sides of a triangle) giving as their resultant a force represented by the third side A D. We might equally well have employed the triangle A C D, by substituting C D for A B.

12. Gravesande's Apparatus.—An apparatus for verifying the parallelogram of forces is represented in Fig. 4. A C D B is a light frame in the form of a parallelogram. A weight P″ can be hung at A, and weights P, P′ can be attached, by means of cords passing over pulleys, to the points B, C. When the weights P, P′, P″ are pro-

portional to A B, A C and A D respectively, the strings attached at B and C will be observed to form prolongations of the sides, and the diagonal A D will be vertical.

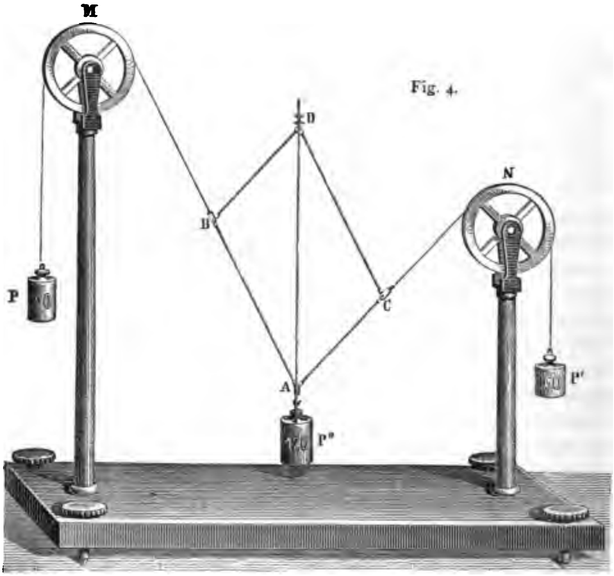
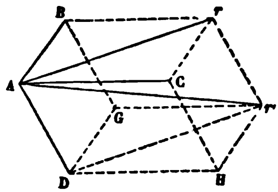


Fig. 4.

13. Resultant of any Number of Forces at a Point.—To find the resultant of any number of forces whose lines of action meet in a point, it is only necessary to draw a crooked line composed of straight lines which represent the several forces. The resultant will be represented by a straight line drawn from the beginning to the end of this crooked line. For by what precedes, if A B C D E be a crooked line such that the straight lines A B, B C, C D, D E represent four forces acting at a point, we may substitute for A B and B C the straight line A C, since this represents their resultant. We may then sub-

stitute A D for A C and C D, and finally A E for A D and D E. One of the most important applications of this construction is to three forces not lying on one plane. In this case the crooked line will consist of three edges of a parallelepiped, and the line which represents the resultant will be the diagonal. This is evident from Fig. 5, in which A B, A C, A D represent three forces acting at A. The resultant of A B and A C is A r, and the resultant of A r and A D is A r'. The crooked line whose parts represent the forces, may be either A B $r r'$, or A B G r', or A D G r', &c., the total number of alternatives being six, since three things can be taken in six different orders. We have here an excellent illustration of the fact that the same final resultant is obtained in whatever order the forces are combined.

Fig. 5.



14. In the examples which we prescribe in illustration of these principles, we shall assume the student to be familiar with the two following geometrical facts:—

1. When the lengths of the sides of a right-angled triangle are numerically expressed, the square of the hypotenuse is equal to the sum of the squares of the other two sides. As a particular case, if these two sides be equal, the hypotenuse is $\sqrt{2}$ times either of them.

2. The angles of an equilateral triangle are each 60° Hence (as appears by bisecting one angle of an equilateral triangle), if one angle of a right-angled triangle be 30°, the side opposite to it is equal to half the hypotenuse.

The student should now work Set I. of Examples.

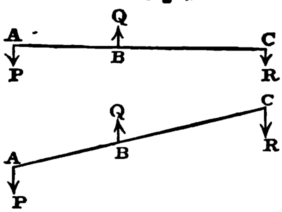
The *inclined plane* affords a good illustration of the resolution of forces. See art. 46, which may here be read with advantage.

15. **Equilibrium of Parallel Forces.**—If three parallel forces, P, Q, R, applied to a rigid body, balance

each other, the following conditions must be fulfilled:—

1. The three lines of action A P, B Q, C R (Fig. 6), must be in one plane.

Fig. 6.



2. The two outside forces P, R, must act in the opposite direction to the middle force Q, and their sum must be equal to Q.

3. The three forces P, Q, R, must be proportional to the three distances BC, A C, A B; in other words, each force must be proportional to the distance between the lines of action of the other two.

16. Principle of Moments.—The student will obtain the clearest ideas on this subject by means of the "principle of moments."

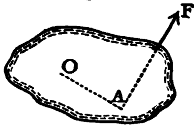
The tendency of a force to produce rotation round an axis (if the directions of the force and axis be at right angles; for example one of them horizontal and the other vertical) is jointly proportional to the force and to the perpendicular distance of its line of action from the axis. This perpendicular distance is often called the "arm" of the force. *Two forces tending to turn a body opposite ways round the axis will balance*, if they are inversely as their arms; in other words, *if one force multiplied by its arm is equal to the other force multiplied by its arm.* The product of a force by its arm is called the "moment" of the force with respect to the axis; and instead of the words "with respect to" the single word "about" or "round" may be used.

The statement which we have italicised is "the principle of moments," and it may be extended to any number of forces by simply adding the moments of those forces which tend to turn the same way.

In a similar sense we speak of the moment of a force about a *point.* The arm, in this case, is the perpendicular

distance between the point and the line of action of the force. In Fig. 7 the moment of the force F about the point O, is F multiplied by O A, and is the same as the moment of F about an axis through O perpendicular to the plane of the diagram.

Fig. 7.



17. To apply the principle of moments to the comparison of any two of the three parallel forces P, Q, R (Fig. 6), we may regard the body as free to turn about one of the three points A, B, C in the upper figure.

The condition of not turning about A is

$$Q \cdot AB = R \cdot AC,$$

which gives the ratio of Q to R.

In like manner, by taking moments about B, we have

$$P \cdot AB = R \cdot BC,$$

and by regarding C as the fixed point

$$P \cdot AC = Q \cdot BC.$$

From any two of these three equations we find

$$\frac{P}{BC} = \frac{Q}{AC} = \frac{R}{AB};$$

that is, the three forces are proportional to the three distances as above stated.

18. The resultant of two parallel forces is of course equal and opposite to the third force which would balance them. To find it, we have first to add the two forces, if they be in the same direction like P and R in Fig. 6, or to subtract the less from the greater if they are in opposite directions like P and Q. We can then determine the distance of the resultant from one of the forces by taking moments; for example if P and Q are given in Fig. 6, we can determine B C by means of the equation

$$P \cdot AB = R \cdot BC,$$

that is $$P \cdot AB = (Q - P) \cdot BC,$$

in which all the quantities are known except B C which is sought.

19. When the distances between the three forces are given and also one of the three forces, the other two forces can be found by taking moments. For example, in Fig. 6 if Q be the given force, P is determined by the equation

$$P \,.\, A\,C = Q \,.\, B\,C.$$

In like manner, when two of the forces are given in magnitude, and one of the distances, we can first find the third force by addition or subtraction, and can then find the other distances by taking moments. For example in Fig. 6 if P, R, and A B are given, we have $Q = P + R$, and A C can then be found by the equation

$$Q \,.\, A\,B = R \,.\, A\,C.$$

The student should now work the first 8 Examples on Parallel Forces.

20. Couple.—There is one case of two parallel forces in opposite directions which requires special attention; that in which the two forces are equal.

When they are nearly equal, supposing them to be the forces P and Q in Fig. 6, their resultant R or $Q - P$ is very small, but the equation of moments

$$P \,.\, A\,B = R \,.\, B\,C$$

still remains true.

Let Q gradually diminish till it becomes equal to P, then while R diminishes to zero B C must increase without limit, for the moment R . B C is to remain unaltered, being always equal to the fixed quantity P . A B.

If we take the total moment of P and Q about *any point* in the plane of the diagram (Q being supposed equal to P), we shall find it to be always the same quantity P . A B, a fact which the student should verify by first taking the point between P and Q, in which case the moments will be the same way round and must be added, and by then taking the point to the left of P or to the

right of Q, in which case the moments will be opposite ways round, and the less must be subtracted from the greater.

21. It thus appears that such a pair of forces applied to a body which is free to turn about a fixed point will tend to make it turn, and that the moment which measures this tendency is the product of one of the two equal forces by the distance between them, quite irrespective of what particular point it is that is fixed.

Such a pair of equal forces is called a *couple*, the distance between them is called the *arm* of the couple, and the constant moment that we have been discussing—the product of one of the two forces by the arm—is called the *moment* of the couple.

A couple is not equivalent to any single resultant force, though it is nearly equivalent to a very small force acting at a very long arm. Hence it cannot be balanced by a single force. It can, however, be balanced by any other couple of equal moment tending to turn the opposite way round.

22. The forces applied to a key to turn it in a lock are a familiar example of a couple; and the forces transmitted by a revolving shaft in machinery are another example; for though, in this case, there are not two prominent forces, as in the other, yet the action is such as could be produced by two equal and opposite forces applied tangentially at opposite extremities of a diameter of the shaft. In fact, the shaft of the key in the former illustration, serves to transmit the couple applied to the handle.

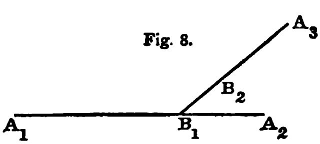
GRAVITY—CENTRE OF GRAVITY.

23. Gravity is the force to which we owe the names "up" and "down." The direction in which gravity acts at any place is called the downward direction, and a line drawn accurately in this direction is called *vertical;* it is the direction assumed by a plumb-line. A plane perpendicular to this direction is called *horizontal*, and is par-

allel to the surface of a liquid at rest. The verticals at different places are not parallel, but are inclined at an angle which is approximately proportional to the distance between the places. As this angle only amounts to 1′ for a distance of a geographical mile, the action of gravity on a body a few feet or a few hundred feet in length, may practically be regarded as consisting of parallel forces.

24. Let A_1 and A_2 (Fig. 8) be any two particles of a rigid body, let w_1 be the weight of the particle A_1, and w_2 the weight of A_2. These weights are parallel forces, and their resultant divides the line A_1 A_2 in the inverse ratio of the forces.

Fig. 8.



As the body is turned about into different positions, the forces w_1 and w_2 remain unchanged in magnitude, and hence the resultant cuts A_1 A_2 always in the same point B_1. This point is called the centre of the parallel forces w_1 and w_2, or the centre of gravity of the two particles A_1 and A_2. The magnitude of the resultant will be $w_1 + w_2$, and we may substitute it for the two forces themselves; in other words, we may suppose the two particles A_1 and A_2 to be collected at their centre of gravity B_1. We can now combine this resultant with the weight w_3 of a third particle A_3, and shall thus obtain a resultant $w_1 + w_2 + w_3$, passing through a definite point B_2 in the line which joins the third particle to the centre of gravity of the first two. The first three particles may now be supposed to be collected at this point, and the same reasoning may be extended until all the particles have been collected at one point. This point will be the *centre of gravity* of the whole body. From the manner in which it has been obtained, it possesses the property that *the resultant of all the forces of gravity on the body passes through it in every position in which the body can be placed.* This property furnishes the most convenient definition of the *centre of gravity.*

25. If the body is homogeneous (that is composed of

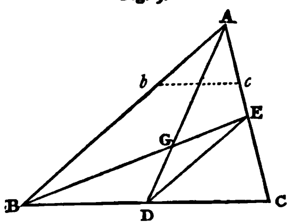
uniform substance throughout), the position of the centre of gravity depends only on the figure, and in this sense it is usual to speak of the centre of gravity of a figure. In like manner, it is customary to speak of the centres of gravity of areas and lines, an area being identified in thought with a thin uniform plate, and a line with a thin uniform wire.

Whenever a homogeneous body contains a point which bisects all lines in the body that can be drawn through it, this point must be the centre of gravity. The centres of a sphere, a circle, a cube, a square, an ellipse, a parallelogram, and a parallelepiped, are examples.

Again when a body consists of a finite number of parts, whose weights and centres of gravity are known, we may regard each part as collected at its own centre of gravity.

26. Centre of Gravity of a Triangle.—To find the centre of gravity of a triangle ABC (Fig. 9), we may begin by supposing it divided into narrow strips by lines (such as *bc*) parallel to BC. It can be shown, by similar triangles, that each of these strips is bisected by the line AD drawn from A to D the middle point of BC. But each strip may be collected at its own centre of gravity, that is at its own middle point; hence the whole triangle may be collected on the line AD; its centre of gravity must therefore be situated upon this line. Similar reasoning shows that it must lie upon the line BE drawn from B to the middle point of AC. It is therefore the intersection of these two lines. If we join DE we can show that the triangles AGB, DGE, are similar, and that

Fig. 9.



$$\frac{AG}{GD} = \frac{AB}{DE} = 2.$$

DG is therefore one-third of DA. The centre of gravity

of a triangle therefore lies upon the line joining any corner to the middle point of the opposite side, and is at one-third of the length of this line from the end where it meets that side.

It is worthy of remark that if three equal particles are placed at the corners of any triangle, they have the same centre of gravity as the triangle. For the two particles at B and C may be collected at the middle point D, and this double particle at D, together with the single particle at A, will have their centre of gravity at G, since G divides DA in the ratio of 1 to 2.

27. Centre of Gravity of a Pyramid.—It can be shown by somewhat similar reasoning that the centre of gravity either of a pyramid or of a cone is on the line drawn from the vertex to the centre of gravity of the base, and is at one-fourth of the length of this line measured from the base.

28. Condition of Standing.—In order that a body may be able to stand on a given base, the vertical drawn through its centre of gravity must fall within the base. If it be supported at three points, as in Fig. 10, the vertical through the centre of gravity must lie within the triangle formed by joining these points, as is evident from considering that if the body toppled over, it would turn round one of the lines of this figure.

Fig. 10.

29. Body supported at one Point.—When a heavy body supported at one point remains at rest, the reaction of the point of support equilibrates the force of gravity. But two forces cannot be in equilibrium unless

they have the same line of action; hence the vertical through the centre of gravity of the body must pass through the point of support. If instead of being supported at a point, the heavy body is supported by an axis about which it is free to turn, the vertical through the centre of gravity must pass through this axis.

30. **Stability.**—Let G be the centre of gravity of a body supported at one point O. Then G may either be vertically above O, as in Fig. 11, or vertically below, as in Fig. 12. In the former case the equilibrium is said to be *unstable*, because if the body be displaced ever so little, it will fall further away; in the latter, the equilibrium is said to be *stable*, because the body when slightly displaced tends to return. If G coincides with O, the equilibrium is called *neutral*. The body, in this case, remains in equilibrium in all positions.

Fig. 11.

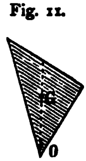
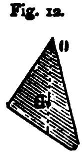


Fig. 12.



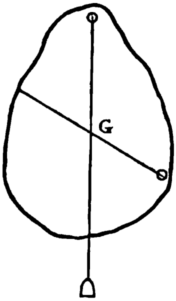
31. More generally when we are speaking of a position of equilibrium of a body subject not only to gravity but to any forces whatever, we call the equilibrium *stable* if the body would return to or oscillate about the position of equilibrium when slightly displaced; *unstable* if the body would not return but move still further away; and *neutral* if it would not move either way, but would remain in equilibrium in the new position.

32. **Experimental Determination of Centre of Gravity.**—In general, if we suspend a body by any point, in such a manner that it is free to turn about this point, it will come to rest in a position of stable equilibrium. The centre of gravity will then be vertically beneath the point of support. If we now suspend the body from another point, the centre of gravity will come vertically beneath this. The centre of gravity will therefore be the intersection of the verticals through the two points of support.

33. To find the centre of gravity of a flat plate or board (Fig. 13), we may suspend it from a point near its

circumference, in such a manner that it sets itself in a vertical plane. Let a plumb-line be at the same time suspended from the same point, and made to leave its trace upon the board by chalking and "snapping" it. Let the board now be suspended from another point, and the operation be repeated. The two chalk lines will intersect each other at that point of the face which is opposite to the centre of gravity; the centre of gravity itself being of course in the substance of the board.

Fig. 13.



34. Tendency of the Centre of Gravity to Descend.—When a body supported at one point O is in unstable equilibrium under the action of gravity, its centre of gravity is in the highest position it can take, as at G (Fig. 14). In stable equilibrium it is at the lowest position it can take, as G (Fig. 15). In both cases, the path of the centre of gravity (if we suppose it to move in a vertical plane) will be a circular arc, FGH, described round O as centre.

Fig. 14.

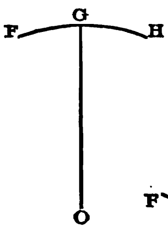
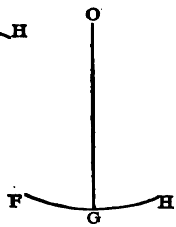


Fig. 15.



If the body is moved away from the position of unstable equilibrium, G descends; if it is moved away from the position of stable equilibrium, G rises. In both cases the direction of motion of G at the instant of passing the position of equilibrium is horizontal.

Very similar remarks apply to a heavy body free to roll upon a supporting surface. The lowest positions of its centre of gravity are positions of stable equilibrium; the highest positions of its centre of gravity are positions of unstable equilibrium; and if it be a sphere or cylinder

resting on a horizontal surface, so that its centre of gravity is always at the same height, all its positions are positions of neutral equilibrium. The centre of gravity always tends to descend, just as it would do if all the weight of the body were collected at the centre of gravity.

WORK.

35. The word "work" is used in dynamics in a special sense.

When a body moves in the direction in which a force tends to urge it, the force is said to *do work* upon the body, and the amount of this force is measured by the product of the force and the distance moved. For example, when a heavy body descends gravity does work upon it. Let w be the body's weight in lbs., and h the vertical distance through which it descends, in feet; then the amount of work done by gravity is wh foot-pounds.

On the other hand, if we lift the body through the vertical distance h we do wh foot-pounds of work against gravity. Work done *against* a force may be regarded as negative work done *by* the force.

When a body moves in a direction perpendicular to that in which a force tends to urge it, no work, either positive or negative, is done by the force; for example, when a body is moved horizontally no work is done upon it by gravity. In computing the work done by or against gravity when a body is moved obliquely downwards or upwards, we are to leave out of account the horizontal component of its motion, and attend only to the vertical component,—in other words, we are to attend only to the difference of levels. The difference of levels multiplied by the weight will be the work done.

36. **Principle of Work.**—In any frictionless machine in which a smaller weight counterbalances a larger, it will be found that when one of these weights descends, and the other accordingly ascends, the vertical distances through which they move are inversely as the weights themselves, so that the amounts of work done—*by* gravity

in the case of one weight, and *against* gravity in the case of the other weight—are equal. If instead of weights we have any two forces balancing one another by means of the machine, the same rule holds; that is to say, if we assist one of the forces and cause it to prevail over the other, the work done by the one force will be equal to the work done against the other. We may express this fact by saying that *a frictionless machine gives out exactly as much work as is put into it.* This fact is called the *principle of work.* The effect of friction is to make the work given out less than the work put in. Much fruitless ingenuity has been employed in attempts to circumvent this law of nature, and contrive a machine which shall give out more work than is put into it. Such contrivances are called "perpetual motions."

MECHANICAL POWERS.

37. We now proceed to a few practical applications of the foregoing principles; and we shall begin with the so-called "mechanical powers," namely, the *lever*, the *wheel and axle*, the *pulley*, the *inclined plane*, the *wedge*, and the *screw*.

38. Problems relating to the **lever** are usually most conveniently solved by taking moments round the fulcrum. The general condition of equilibrium is, that the moments of the power and the weight about the fulcrum must be in opposite directions, and must be equal. When the power and weight act in parallel directions, the conditions of equilibrium are precisely those of three parallel forces (art. 15), the third force being the reaction of the fulcrum.

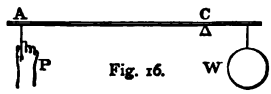


Fig. 16.

It is usual to distinguish three "orders" of lever. In levers of the first order (Fig. 16) the fulcrum is between the power and the weight. In those of the second order (Fig. 17) the weight is between the power and the fulcrum.

In those of the third order (Fig. 18) the power is between the weight and the fulcrum.

In levers of the second order, (supposing the forces

Fig. 17.

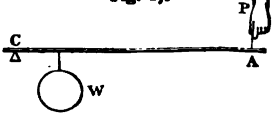
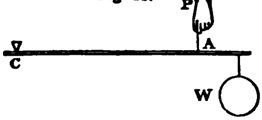


Fig. 18.



parallel,) the weight is equal to the sum of the power and the pressure on the fulcrum; and in levers of the third order, the power is equal to the sum of the weight and the pressure on the fulcrum; since the middle one of three parallel forces in equilibrium must always be equal to the sum of the other two.

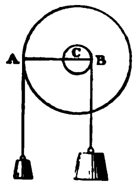
39. The *arms of a lever* are the two portions of it intermediate, respectively, between the fulcrum and the power, and between the fulcrum and the weight. If the lever is bent, or if, though straight, it is not at right angles to the lines of action of the power and weight, it is necessary to distinguish between the arms of the lever as above defined (which are parts of the lever), and the *arms of the power and weight* regarded as forces which have moments round the fulcrum. In this latter sense, (which is always to be understood unless the contrary is evidently intended,) the arms are the perpendiculars dropped from the fulcrum upon the lines of action of the power and weight.

40. In the above statements of the conditions of equilibrium, we have neglected the weight of the lever itself. To take this into account, we have only to suppose the whole weight of the lever collected at its centre of gravity, and then take its moment round the fulcrum. We shall thus have three moments to take account of, and the sum of the two that tend to turn the lever one way, must be equal to the one that tends to turn it the opposite way.

41. Every machine when in action serves to transmit

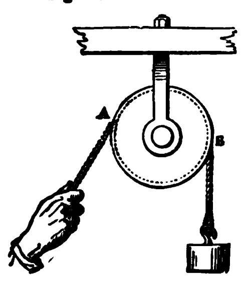
work without altering its amount; but the *force* which the machine gives out (equal and opposite to what is commonly called the *weight*) may be much greater or much less than that by which it is driven (commonly called the *power*). When it is greater, the machine is said to confer *mechanical advantage*, and the mechanical advantage is measured by the ratio of the weight to the power for equilibrium. In the lever, when the power has a longer arm than the weight, the mechanical advantage is equal to the quotient of the longer arm by the shorter.

Fig. 19.



42. **The wheel and axle** (Fig. 19) may be regarded as an endless lever. The condition of equilibrium is at once given by taking moments round the common axis of the wheel and axle (art. 16). If we neglect the thickness of the ropes, the condition is that the power multiplied by the radius of the wheel must equal the weight multiplied by the radius of the axle; but it is more exact to regard the lines of action of the two forces as coinciding with the axes of the two ropes, so that each of the two radii should be increased by half the thickness of its own rope. If we neglect the thickness of the ropes, the mechanical advantage is the quotient of the radius of the wheel by the radius of the axle.

Fig. 20.



43. **A pulley**, when fixed in such a way that it can only turn about a fixed axis (Fig. 20), confers no mechanical advantage. It may be regarded as an endless lever of the first order with its two arms equal.

The arrangement represented in Fig. 21 gives a mechanical advantage of 2; for the lower or movable pulley may be regarded as an endless lever of the second

order, in which the arm of the power is the diameter of the pulley, and the arm of the weight is half the diameter. The same result is obtained by employing the principle of work; for if the weight rises 1 inch, 2 inches of slack are given over, and therefore the power descends 2 inches.

44. In Fig. 22 there are six pulleys, three at the upper and three at the lower block, and one cord passes round them all. All portions of this cord (neglecting friction)

Fig. 21.

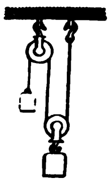


Fig. 22.

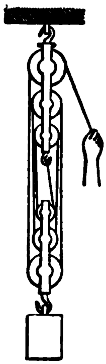
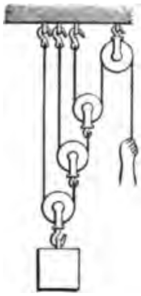


Fig. 23.



are stretched with the same force, which is equal to the power; and six of these portions, parallel to one another, support the weight. The power is therefore one-sixth of the weight, or the mechanical advantage is 6.

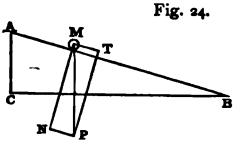
45. In the arrangement represented in Fig. 23 there are three movable pulleys, each hanging by a separate cord. The cord which supports the lowest pulley is stretched with a force equal to half the weight, since its two parallel portions jointly support the weight. The cord which supports the next pulley is stretched with a force half of this, or a quarter of the weight; and the next cord with a force half of this, or an eighth of the weight; but this cord is directly attached to the power. Thus the

power is an eighth of the weight, or the mechanical advantage is 8. If the weight and the block[1] to which it is attached rise 1 inch, the next block rises 2 inches, the next 4, and the power moves through 8 inches. Thus, the work done by the power is equal to the work done upon the weight.

In all this reasoning we neglect the weights of the blocks themselves; but it is not difficult to take them into account when necessary.

46. We now come to the **inclined plane.** Let A B (Fig. 24) be any portion of such a plane, and let A C and B C be drawn vertically and horizontally. Then A B is called the *length*, A C the *height*, and C B the *base* of the inclined plane. The force of gravity upon a heavy body M resting on the plane, may be represented by a vertical line M P, and may be resolved by the parallelogram of forces (art. 11) into two components, M T, M N, the former parallel and the latter perpendicular to the plane.

Fig. 24.



A force equal and opposite to the component M T will suffice to prevent the body from slipping down the plane. Hence, if the power act parallel to the plane, and the weight be that of a heavy body resting on the plane, the power is to the weight as M T to M P; but the two triangles M T P and A C B are similar, since the angles at M and A are equal, and the angles at T and C are right angles; hence M T is to M P as A C to A B, that is, as the height to the length of the plane.

47. The investigation is rather easier by the principle of work (art. 36). The work done by the power in drawing the heavy body up the plane, is equal to the power multiplied by the length of the plane. But the work done upon the weight is equal to the weight multi-

[1] The "pulley" is the revolving wheel. The pulley, together with the frame in which it is inclosed, constitute the "block."

plied by the height through which it is raised, that is, by the height of the plane. Hence we have

Power × length of plane = weight × height of plane; or
power : weight :: height of plane : length of plane.

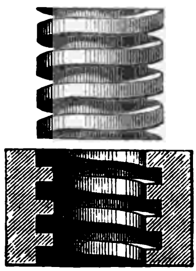
48. If, instead of acting parallel to the plane, the power acted parallel to the base, the work done by the power would be the product of the power by the base; and this must be equal to the product of the weight by the height; so that in this case the condition of equilibrium would be

Power : weight :: height of plane : base of plane.

49. In these investigations we have neglected friction. The **wedge** may be regarded as a case of the inclined plane; but its practical action depends to such a large extent upon friction and impact[1] that we cannot profitably discuss it here.

50. The **screw** (Fig. 25) is also a case of the inclined plane. The length of one convolution of the thread is the length of the corresponding inclined plane, the step of the screw, or distance between two successive convolutions (measured parallel to the axis of the screw), is the height of the plane, and the circumference of the screw is the base of the plane. This is easily shown by cutting out a right-angled triangle in paper, and bending it in cylindrical fashion so that its base forms a circle.

Fig. 25

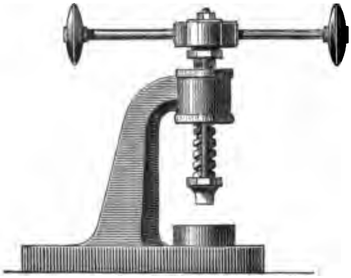


51. In the screw press (Fig. 26) the screw is turned by means of a lever, which gives a great increase of mechani-

[1] An *impact* (for example a blow of a hammer) may be regarded as a very great (and variable) force acting for a very short time. The magnitude of an impact is measured by the momentum which it generates in the body struck.

cal advantage. In one complete revolution, the pressures applied to the two handles of the lever to turn it, do work equal to their sum multiplied by the circumference of the circle described (approximately) by either handle (we suppose the two handles to be equidistant from the axis of revolution); and the work given out by the machine, supposing the resistance at its lower end to be constant, is equal to this resistance multiplied by the distance between the threads. These two products must be equal, friction being neglected.

Fig. 26.



BALANCE.

52. In the common *balance* (Fig. 27) there is a stiff piece of metal, AB, called the *beam*, which turns about the sharp edge O of a steel wedge forming part of the beam and resting upon two hard and smooth supports. There are two other steel wedges at A and B, with their edges upwards, and upon these edges rest the hooks for supporting the scale pans. The three edges (called knife-edges) are parallel to one another and perpendicular to the length of the beam, and are very nearly in one plane.

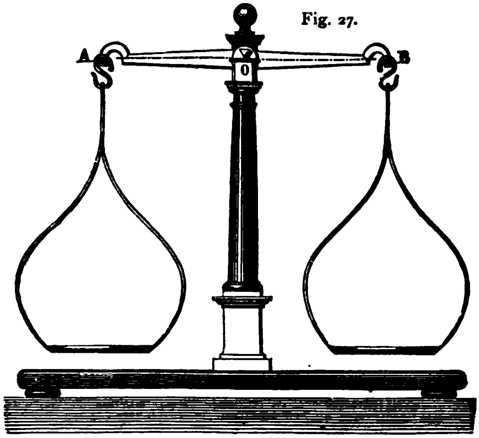
53. The qualities requisite in a balance are:

1. That it be consistent with itself; that is, that it shall give the same result in successive weighings of the same body. This depends chiefly on the trueness of the knife-edges.

2. That it be just. This requires that the distances AO, OB, be equal, and also that the beam remain horizontal when the pans are empty. Any inequality in

the distances A O, O B, can be detected by putting apparently equal weights into the two pans, and then inter-

Fig. 27.



changing these weights. If they still balance each other, the arms are equal.

3. Delicacy or sensibility (that is, the power of indicating inequality between two weights even when their difference is very small).

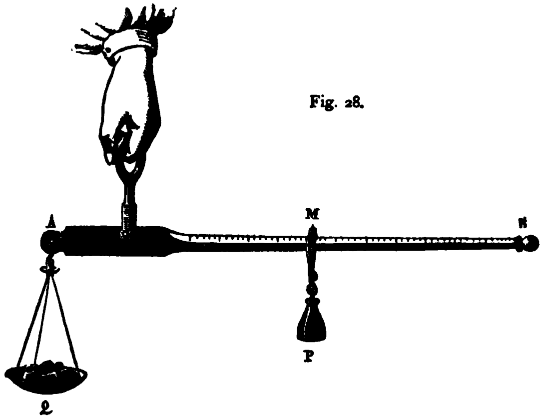
This requires a minimum of friction, and a very near approach to neutral equilibrium (art. 30). In absolutely neutral equilibrium, the smallest conceivable force is sufficient to produce a displacement to the full limit of neutrality; and in barely stable equilibrium a small force produces a large displacement. The condition of stability is that if the weights supported at A and B be supposed collected at these edges, the centre of gravity of the system composed of the beam and these two weights shall be below the middle edge O. The equilibrium would be neutral if this centre of gravity exactly coincided with O; and it is necessary as a condition of delicacy that its distance below O be very small.

4. Facility for weighing quickly is desirable, but must sometimes be sacrificed when extreme accuracy is required.

The delicate balances used in chemical analysis are provided with a long pointer attached to the beam. The end of this pointer moves along a graduated arc as the beam vibrates; and if the weights in the two pans are equal, the excursions of the pointer on opposite sides of the zero point of this arc will also be equal. Much time is consumed in watching these vibrations, as they are very slow; and the more nearly the equilibrium approaches to neutrality, the slower they are. Hence quick weighing and exact weighing are to a certain extent incompatible.

54. **Double Weighing.**—Even if a balance be not just, yet if it be consistent with itself, a correct weighing can be made with it in the following manner:—Put the body to be weighed in one pan, and counterbalance it with sand or other suitable material in the other. Then

Fig. 28.



remove the body and put in its place such weights as are just sufficient to counterpoise the sand. These weights are evidently equal to the weight of the body. This process is called *double weighing*, and is often employed

(even with the best balances) when the greatest possible accuracy is desired.

55. Steelyard.—The steelyard (Fig. 28) is an instrument for weighing bodies by means of a single weight, P, which can be hung at any point of a graduated arm O B. As P is moved further from the fulcrum O, its moment round O increases, and therefore the weight which must be hung from the fixed point A to counterbalance it increases. Moreover, equal movements of P along the arm produce equal additions to its moment, and equal additions to the weight at A produce equal additions to the opposing moment. Hence the divisions on the arm (which indicate the weight in the pan at A) must be equidistant.

FIRST PRINCIPLES OF KINETICS.

56. Principle of Inertia.—A body not acted on by any forces, or only acted on by forces which are in equilibrium, will not commence to move; and if it be already in motion with a movement of pure translation,[1] it will continue its velocity of translation unchanged, so that each of its points will move in a straight line with uniform velocity. This is Newton's first law of motion, and is stated by him in the following terms:—

"Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled by impressed forces to change that state."

It is often called the *principle of inertia*. The tendency to continue in a state of rest is manifest to the most superficial observation. The tendency to continue in a state of uniform motion can be inferred from such facts as the long continuance of the motion of a pendulum when once set oscillating, or the great distance that a heavy ball will roll on a smooth surface of ice. The gradual destruction of the motion in these cases is suffi-

[1] A body is said to have a movement of pure translation, if all points of it have equal and parallel velocities. Every motion of a solid body is either a translation or a rotation or a compound of the two.

ciently accounted for by the slight opposing forces of friction. The motions of the earth and planets in their orbits show no trace whatever of frictional retardation, but continue with undiminished velocity from century to century.

57. The tendency to continue in motion is the cause of the effects which are observed when a carriage or railway train is suddenly stopped. The passengers are thrown forward in virtue of the velocity with which they were moving before the stoppage occurred. The same principle explains the falls which are often received in alighting from a carriage in motion; the whole body has a forward motion, and when the feet are held back by coming in contact with the ground, the upper part of the body continues to move forward. It is by taking advantage of the inertia of matter that we are able to fix the head of a hammer on the handle by striking the end of the handle on the ground. The handle is suddenly stopped by the collision, and the head continues to move for a short distance in spite of the powerful resistances which oppose it.

58. **Second Law. Acceleration.**—Newton's second law of motion relates to the change of motion produced in a body by a force acting on it. We shall for the present confine ourselves to the case of rectilinear motion.

When a body originally at rest is acted on by a force which remains constant both in magnitude and direction, it moves in a straight line with continually increasing velocity. The straight line in which it moves is identical with the direction of the force, and its velocity increases exactly in proportion to the time that the motion has lasted. The velocity produced in the unit of time—say in one second—is called the *acceleration* of the motion.

59. **Mass.**—The magnitude of the acceleration depends partly on the force and partly on the body. For the same body a double force would produce a double acceleration; and for the same force doubling the body would halve the acceleration. The quantity of matter in a body of given material can therefore be determined from knowing the

force and the acceleration. Bodies, whether of the same or different materials, are said to be "equal in mass" if they receive equal accelerations when acted on by equal forces; or the bodies themselves may be called "equal masses."

60. Third Law of Motion. Action and Reaction.—Forces always occur in pairs, every exertion of force being a mutual action between two bodies. Whenever a body is acted on by a force, the body from which this force proceeds is acted on by an equal and opposite force. The earth attracts the moon, and the moon attracts the earth. A magnet attracts iron and is attracted by iron. When two boats are floating freely, a rope attached to one and hauled in by a person in the other, makes each boat move towards the other. The velocities thus produced in the two bodies are inversely as their masses, and if we agree to give the name "momentum" to mass multiplied by velocity, then the momenta produced in the two bodies by their mutual action are equal. For example, if a gun is freely suspended so that there is no force opposing its recoil, the momentum of its recoil will be equal to the momentum of the ball which is fired from it.

LAWS OF FALLING BODIES.

61. In air, a slip of paper or gold-leaf falls slowly on account of the large surface which it exposes to the friction of the air in proportion to its weight. In a perfect vacuum all bodies would fall equally fast. This is approximately proved by the well-known "guinea-and-feather" experiment. A glass tube, some four feet long, contains a feather and a coin, or any two bodies which would fall with very unequal velocities in air. There is a stopcock at one end of the tube for fixing it on the air-pump; and when the air has been nearly all exhausted the feather falls nearly as fast as the coin.

62. Mass Proportional to Weight.—It follows that bodies which are equal in weight are also equal in

mass (art. 59), for when allowed to fall in vacuo they are acted on by equal forces—namely their weights—and they receive equal accelerations.

Newton tested this point still more severely by experiments with pendulums (*Principia*, book III. prop. vi.). He procured two round wooden boxes of the same size and weight, and suspended them by threads eleven feet long. One of them he filled with wood, and he placed very accurately in the centre of oscillation of the other the same weight of gold. The boxes hung side by side, and, when set swinging in equal oscillations, went and returned together for a very long time. Here the forces concerned in producing and checking the motion, namely, the force of gravity and the resistance of the air, were the same for the two pendulums, and as the movements produced were the same, it follows that the masses were equal. Newton remarks that a difference of mass amounting to a thousandth part of the whole could not have escaped detection. He experimented in the same way with silver, lead, glass, sand, salt, water, and wheat, and with the same result. He therefore infers that universally bodies of equal mass gravitate equally towards the earth at the same place. He further extends the same law to gravitation generally, and establishes the conclusion that the mutual gravitating force between any two bodies depends only on their masses and distances, and is independent of their materials.

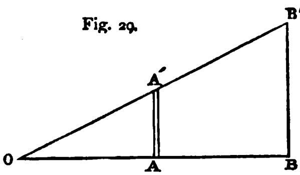
The time of revolution of the moon round the earth, considered in conjunction with her distance from the earth, shows that the relation between mass and gravitation is the same for the material of which the moon is composed as for terrestrial matter; and the same conclusion is proved for the planets by the relation which exists between their distances from the sun and their times of revolution in their orbits.

63 **Velocity Acquired.**—The velocity acquired in one second by a body falling in vacuo at any part of the earth's surface is rather more than 32 ft. per second.

The exact value is usually denoted by the letter *g*. In working such examples as we shall prescribe the student may take *g* as 32. The velocity acquired in half a second will be ½ *g* or 16. In two seconds it will be 2 *g* or 64, and in *t* seconds it will be *g t*.

64. Space Described.—To find the space described it is convenient to use a geometrical illustration. The distance described by a body moving with uniform velocity is calculated by multiplying the velocity by the time; just as the area of a rectangle is calculated by multiplying its length by its breadth. Hence, if we draw a line O A′ B′ such that its ordinates A A′, B B′, &c., drawn vertically represent the velocities with which a body is moving at the times represented by O A, O B, &c., drawn horizontally, it can be shown that the whole distance described in the time O B is represented by the area O B′ B. In fact, this area can be divided into narrow strips (one of which is shown at A A′, Fig. 29), each of which may practically be regarded as a rectangle, whose height represents the velocity with which the body is moving during the very small interval of time represented by its base, and whose area therefore represents the distance described in this time.

Fig. 29.



This would be true for the distance described by a body moving from rest with any law of velocity. In the case of falling bodies the law is that the velocity is simply proportional to the time; hence the ordinates A A′, B B′, &c., must be directly as the abscissæ O A, O B; this proves that the line O A′ B′ must be straight; and the figure O B′ B is therefore a triangle. Its area will be half the product of O B and B B′. But O B represents the time *t* occupied by the motion, and B B′ the velocity *g t* at the end of this time. The area of the triangle therefore represents half the product of *t* and *g t*, that is, represents

$\frac{1}{2} g t^2$, which is accordingly the distance described in the time t. Denoting this distance by s, and the velocity at the end of time t by v, we have thus the two formulæ

$$\left.\begin{aligned} v &= gt \qquad (1) \\ s &= \tfrac{1}{2} g t^2, \qquad (2) \end{aligned}\right\}$$

from which we easily deduce

$$v^2 = 2gs. \qquad (3)$$

These three formulæ are of great importance, and should be committed to memory by the student. When g is given, and any one of the three quantities t, v, and s, they enable us to find the other two. The first six examples on falling bodies should now be worked. The following hints will be useful.

65. Questions referring to velocity and time and not to distance should be worked by formula (1).

Questions referring to distance and time and not to velocity should be worked by (2).

Questions referring to distance and velocity and not to time should be worked by (3).

Again, when a body is thrown vertically upwards and falls back to the point from which it was thrown, the return journey is an exact copy, reversed, of the upward journey. Thus the time of descent is equal to the time of ascent, and the velocity at each height is the same in going up as in coming down.

The time of rising to the highest point when the initial velocity is given can therefore be found by formula (1), and the height of the highest point by (3).

66. Effect of Initial Velocity.—When the motion discussed does not begin or end at the highest point, the three formulæ must be supplemented in the following way.

To find the actual velocity at the time t, subtract the velocity given by (1) from the initial upward velocity. If the remainder is positive the velocity is upward; if negative, downward.

To find the actual height of the body above the point

of projection at time t, subtract the distance given by (2) from Vt, where V denotes the initial upward velocity. Vt is clearly the height that would have been attained if there had been no gravity, but the body had continued to move uniformly. The modification required in these rules when the initial velocity is downward instead of upward may be left to the student.

67. **Atwood's Machine.**—If two equal masses, each of magnitude M, are hung from the ends of a fine cord passing over a pulley which turns without friction, they will of course balance one another. If initially at rest they will remain at rest, and if initially in motion (one descending and the other ascending vertically) they will continue to move with uniform velocity. If an additional mass be laid upon one of them, this one will descend with continually increasing speed and draw the other up. The acceleration will be much less than g, for the weight of the mass m has to move not only this mass but the other two as well. The whole mass thus moved is $2M+m$, and the acceleration will be the same fraction of g that m is of $2M+m$. In other words, denoting the acceleration by f we have

$$f=\frac{m}{2M+m}g.$$

The formulæ for the motion will be

$$\begin{aligned} v &= ft \\ s &= \tfrac{1}{2}ft^2 \\ v^2 &= 2fs. \end{aligned}$$

This is on the assumption that friction and the masses of the pulley and cord can be neglected.

A machine constructed on this principle (called Atwood's machine) is very much used for illustrating the laws of falling bodies, whose motion it imitates on a much slower scale. The additional mass m is of such a shape that, when laid on one of the equal masses, M, it projects far beyond it on both sides. The mass M thus loaded descends in front of a scale of equal parts; and at any

point we please of this scale a ring is fixed, which is large enough for the mass M to pass through it, but intercepts the additional mass. From this point onward the velocity is uniform, so that the velocity attained at the instant of arriving at the ring can be determined by observing the space described in a measured time after passing it.

68. Acceleration.—Such motion as that of one of the masses in Atwood's machine, or of a body falling freely, is said to be *uniformly accelerated*. This designation means that the velocity is increased by equal amounts in all equal intervals of time. A body acted on by a constant force in the line in which it is travelling, always moves with uniform acceleration. This is an obvious inference from the second law of motion, which asserts that the acceleration of a body is inversely as its mass and directly as the force acting on it. Increasing or diminishing acceleration in a given mass is the evidence of increasing or diminishing force, and uniform acceleration is the evidence of uniform force.

69. Resistance of the Air.—The formulæ above given for the motion of falling bodies are rigorously applicable to motion in vacuo, and are sensibly correct for motion in air as long as the resistance of the air is insignificant in comparison with the force of gravity. The force of gravity upon a body is the same at all velocities; but the resistance of the air increases with the velocity, and increases more and more rapidly as the velocity becomes greater; so that while at very slow velocities an increase of 1 per cent. in velocity would give an increase of 1 per cent. in the resistance, at a higher velocity it would give an increase of 2 per cent., and at the velocity of a cannon-ball an increase of 3 per cent.[1] The formulæ are therefore sensibly in error for high velocities. They are also in error for bodies which, like feathers or gold-leaf, have a large surface in proportion to their weight.

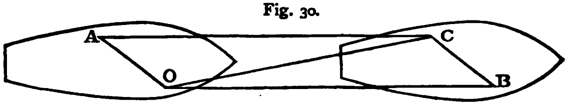
[1] This is only another way of saying that the resistance varies approximately as the velocity when very small, and approximately as the cube of the velocity for velocities like that of a cannon-ball.

COMPOSITION OF MOTIONS.

70. When a boat is sailing on a river, the motion of the boat relative to the shore is compounded of its motion relative to the water and the motion of the water relative to the shore. If a person is walking along the deck of the boat in any direction, his motion relative to the shore is compounded of three motions, namely the two above mentioned and his motion relative to the boat.

Let X, Y, and Z be any three bodies or systems. The motion of X relative to Y, compounded with the motion of Y relative to Z, is the motion of X relative to Z. This is to be taken as the definition of what is meant by compounding two motions; and it leads very directly to the result that two rectilinear motions are compounded by the parallelogram law. For if a body moves along the deck of a ship from O to A (Fig. 30), and the ship in the

Fig. 30.

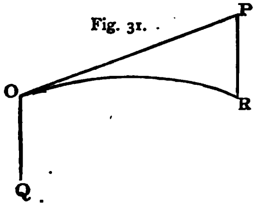


meantime advances through the distance O B, it is obvious that, if we complete the parallelogram O B C A, the point A of the ship will be brought to C, and the movement of the body in space will be from O to C. If the motion along O A is uniform, and the motion of the ship is also uniform, the motion of the body through space will be a uniform motion along the diagonal O C. Hence, *if two component velocities be represented by two lines drawn from a point, and a parallelogram be constructed on these lines, its diagonal will represent the resultant velocity.*

It is obvious that if O A in the figure represented the velocity of the ship and O B the velocity of the body relative to the ship, we should obtain the same resultant velocity O C. This is a general law; the interchanging of velocities which are to be compounded does not affect their resultant.

71. Projectiles.—If, instead of being simply let fall, a body is projected in any direction, its motion will be compounded of the motion of a falling body and a uniform motion in the direction of projection. Thus if O P (Fig. 31) is the direction of projection, and O Q the vertical through the point of projection, the body would move along O P keeping its original velocity unchanged, if it were not disturbed by gravity. To find where the body will be at any time t, we must lay off a length O P equal to V t, V denoting the velocity of projection, and must then draw from P the vertical line P R downwards equal to $\frac{1}{2} g t^2$, which is the distance that the body would have fallen in the time if simply dropped. The point R thus determined, will be the actual position of the body. The velocity of the body at any time will in like manner be found by compounding the initial velocity with the velocity which a falling body would have acquired in the time.

Fig. 31.



The path of the body will be a curve, as represented in the figure, O P being a tangent to it at O, and its concavity being downwards. The equations above given, namely

$$O P = V t, \qquad P R = \tfrac{1}{2} g t^2,$$

show that P R varies as the square of O P, and hence that the path (or *trajectory* as it is technically called) is a parabola, whose axis is vertical.

72. Centrifugal Force.—Whenever a body moves in a curved path there must be some force urging it towards the concave side. If its path be circular and its motion uniform, this force acts directly towards the centre of the circle, and the reaction of the inertia of the body against the string or other instrument by which the force is applied, is called *centrifugal force.*

73. Energy.—When work has been done in lifting a weight, the same amount of work may be obtained from the weight by letting it drive a machine as it runs down. The winding up of a clock driven by weights is an example; and when we wind up a clock driven by a spring the principle is the same, though the forces concerned depend on elasticity and not on gravity. A body which resembles a wound-up clock in being ready to do work by letting its parts run down into the position which they tend to take, is said to contain *potential energy* or *statical energy*. "Energy," in dynamics and in physics generally, means capability of doing work, and it is measured in the same units as work, for example in foot-pounds. The water in a mill-dam is an excellent example of potential energy.

74. Energy of Motion.—When work is done, not in overcoming gravity or any opposing *force*, but in overcoming *inertia*, the mass acted on acquires velocity, and it can be shown that the work done is proportional to the mass and to the square of the velocity. The student can easily verify this in the case of gravity doing work upon a body which is let fall, for the force in this case, being the weight of the body, is proportional to its mass, and formula (3) for falling bodies shows that the distance is proportional to the square of the velocity.

In this case also, the body acted on acquires power to do work; for example a cannon ball can penetrate obstacles which oppose very great resistance, or if fired directly upwards can carry itself to a great height in the air. The work done by a blow of a hammer is another example. A moving body is therefore to be regarded as possessing energy in virtue of its motion. This is called *kinetic energy* or energy of motion. It must not be confounded with momentum; for momentum is proportional to *velocity*, whereas *kinetic energy* is proportional to the square of velocity. If a gun perfectly free to recoil is 100 times as heavy as the ball which is fired from it, the kinetic energy of the ball will be 100 times that of the recoiling gun, but their momenta will be equal. The

same force, due to the powder, acts upon both for the same time, but through very different distances; for while the ball is travelling the length of the barrel, the gun recoils through only one hundredth part of this distance.

PENDULUM.

75. When a body slides under the action of its own weight down an incline without friction, it acquires the same velocity as if it had fallen vertically through the same difference of levels. For example, if the incline be an inclined *plane* the velocity acquired is the same as in falling down the height of the plane. This is an instance of the general law that, when there is no friction, the energy of motion gained is equal to the work done by the forces which produce the motion. The velocity which a body acquires in sliding down an incline is just sufficient to carry it up an opposite incline of the same total height, still supposing that there is no friction. This principle is illustrated by the pendulum.

76. Any heavy rigid body, oscillating under the action of gravity about a horizontal axis, may be regarded as a pendulum.

The motion of such a body is similar to that of a heavy ball suspended by a thread; and in order to simplify the circumstances as much as possible, it is usual to begin by discussing the imaginary case of a very small heavy particle, suspended by a perfectly flexible and inextensible thread so fine that its mass may be neglected. This imaginary arrangement is called a *simple pendulum;* and in contradistinction from it, a rigid body oscillating about an axis is called a *compound pendulum.*

77. Let A B (Fig. 32) represent a simple pendulum, A being the heavy particle, and A B the thread, attached to the fixed point B. The path of A will be the circular arc A C A′ described round B as centre, and in the position of equilibrium A will be at C, the lowest point of this arc, which is vertically under B. The force of gravity upon

A, in the position represented in the figure, can be resolved (as shown by the dotted parallelogram) into two forces, one acting in the direction of the thread produced, and the other along a tangent to the arc. The former component simply tightens the thread, the latter governs the motion of the particle, and varies nearly in direct proportion to the arc C A as long as the arc does not exceed 2 or 3 degrees.

Fig. 32.

78. If we suppose the particle to be let go in the position represented in the figure, it will descend under the action of gravity, and will gain in energy of motion what it loses in energy of position. The work done by gravity during the descent from A to C is the weight of the particle multiplied by the vertical distance D C; and the same amount of work is done against gravity in the ascent on the other side, during which the energy of motion is reconverted into energy of position. We here neglect the resistance of the air and every kind of friction.

79. The arc A C or its equal C A′ is called the *amplitude*.

The time from A to A′ and back again is called the *period* or the *periodic time*. It is also sometimes called the time of a *double* or *complete* oscillation. It is more usual to speak of the time from A to A′, which is called for distinction the time of a *single* oscillation.[1]

The time of an oscillation is nearly the same for all amplitudes from the smallest visible up to 5° or 10°. This property is called the *isochronism* of the pendulum

[1] The word *vibration* may be used instead of *oscillation*.

(from ἴσος, equal, and χρονος, time). The small vibrations of any vibrating body—for example, a spring, a tuning-fork, or a musical string, or the needle of a mariner's compass—are isochronous, as well as those of a pendulum.

80. The time T of a *single* oscillation is

$$T = \pi \sqrt{\frac{l}{g}},$$

l denoting the length of the pendulum, g the acceleration of a freely falling body, and π the ratio of the circumference of a circle to the diameter. For example, to find the length of a pendulum that beats seconds we must put $T = 1$, and we shall obtain $l = \frac{g}{\pi^2}$, which if we take g as 32·2 and π as 3·14 gives $l = \frac{32{\cdot}2}{9{\cdot}86} = 3{\cdot}27$ ft.

A pendulum to beat every two seconds must have four times this length, or about 13 ft.; for the formula shows that T is proportional to the square root of l.

The former also shows that T is inversely proportional to the square root of g. Hence the number of vibrations made in a given time is directly as the square root of g, and g is therefore proportional to the square of the number of vibrations. It is by comparing the number of vibrations made by the same pendulum at different places that the variation of gravity over the surface of the earth has been measured.

Every actual pendulum keeps time with a simple pendulum of appropriate length (called the *equivalent simple pendulum*), and it is this length that is usually meant when the "length of a pendulum" is spoken of.[1]

81. Calculation of Energy of Motion.—In order to calculate the number of foot-pounds of energy in a moving body we may proceed as follows.

Suppose a body weighing m lbs. to fall through s feet.

[1] That point of a pendulum, in a line through the point of suspension and centre of gravity, at which the whole mass could be collected without altering the time of vibration, is called the *centre of oscillation*.

The work done upon it is $m s$ foot-pounds. But by the third formula for falling bodies

$$v^2 = 2 g s \therefore s = \frac{v^2}{2g},$$

and $$m s = \frac{m v^2}{2g} = \frac{m v^2}{64} = m\left(\frac{v}{8}\right)^2.$$

Hence if we take one-eighth part of the velocity in feet per second, square it, and multiply by the mass in lbs., we shall obtain very approximately the energy of the body's motion in foot-pounds.

FRICTION.

82. When two bodies are pressed together in such a manner that the direction of their mutual pressure is not normal to the surface of contact, the pressure can be resolved into two parts, one normal and the other tangential. The tangential component is called the *force of friction* between the two bodies. The friction is called *kinetical* or *statical* according as the bodies are or are not sliding one upon the other.

As regards kinetical friction, experiment shows that if the normal pressure between two given surfaces be changed, the tangential force changes almost exactly in the same proportion; in other words, the ratio of the tangential to the normal force is nearly constant for two given surfaces. This ratio is called the *coefficient of kinetical friction* between the two surfaces.

83. The statical friction between two given surfaces is zero when their mutual pressure is normal, and increases with the obliquity of the pressure. This obliquity cannot increase beyond a certain limit, depending on the nature of the bodies, and seldom amounting to so much as 45°. The limiting obliquity—that is, the greatest angle that the mutual force can make with the normal—is called the *limiting angle of friction*, or the *angle of sliding*, for the two surfaces; and the ratio of the tangential to the normal

component when the mutual force acts at the limiting angle, is called the *coefficient of statical friction* for the two surfaces. The coefficient and limiting angle remain nearly constant when the intensity of the pressure is varied.

The coefficient of statical friction is in almost every case greater than the coefficient of kinetical friction; in other words, friction offers more resistance to the commencement of sliding than to the continuance of it.

A body which has small coefficients of friction with other bodies is called slippery.

84. When an inclined plane is gradually tilted till a body lying on it slides under the action of gravity, the inclination of the plane at which sliding begins is equal to the limiting angle of friction as above defined; for the mutual pressure is vertical, and its inclination to the normal is therefore the same as the inclination of the plane to the horizon. Hence in the triangle M N P (Fig. 24), M N represents the normal pressure and N P the tangential force, or force of friction. But this triangle is similar to the triangle B C A; hence the ratio of the friction to the normal pressure is the ratio of A C to C B, that is, the ratio of the height of the inclined plane to its base.

Again, if the inclination of a plane be such that the motion of a body sliding down it under the action of gravity is neither accelerated nor retarded, the ratio of the height to the base will be the coefficient of kinetical friction.

EXAMPLES IN DYNAMICS.

I.—Forces Acting at a Point. Arts. 9–14.

1. Two forces of 6 and 8 units act upon a body in lines which meet in a point and are at right angles. Find the magnitude of their resultant.

2. Two forces of 100 lbs. each act upon a body in lines which meet at a point and are at right angles. Find the magnitude of their resultant.

3. The resultant of two forces at right angles is 13 lbs., and one of the two forces is 5 lbs. Find the other.

4. A 12-lb. weight hangs by a string. A second string is tied to the weight and pulled horizontally with a force of 5 lbs. Draw a figure showing how much the first string will be deflected from the vertical, and calculate the tension which the first string will now have to bear.

5. A weight of 20 lbs., hanging as in the last question, is drawn aside by a horizontal string till the deflection is 45°. What will now be the tensions in the two strings?

6. If it be drawn aside till the deflection is 60°, what will be the tension in the original string?

7. What will be the tension in the horizontal string?

8. If the triangle of forces for three strings whose pulls equilibrate each other be an equilateral triangle, what angles will the strings make with each other?

9. Two equal forces act at an angle of 120°. Find their resultant in magnitude and direction.

10. Show that if the sides of a triangle are *perpendicular* to three forces in equilibrium, the sides represent the forces in magnitude. (Suppose the triangle turned through a right angle.)

11. A point is urged to the east by a force of 3 lbs., to

the north by a force of 4 lbs., and vertically downwards by a force of 12 lbs. Find the total horizontal component and the total force.

II.—Parallel Forces. Arts. 15–22.

1. A straight rod 10 ft. long is supported at a point 3 ft. from one end. What weight hung from this end will be supported by 12 lbs. hung from the other, the weight of the rod being neglected?

2. Weights of 15 and 20 lbs. are hung from the two ends of a straight rod 70 in. long. Find the point about which the rod will balance, its own weight being neglected.

3. A weight of 100 lbs. is slung from a pole which rests on the shoulders of two men, A and B. The distance between the points where the pole presses their shoulders is 10 ft., and the point where the weight is slung is 4 ft. from the point where the pole presses on A's shoulder. Find the weight borne by each, the weight of the pole being neglected.

4. A uniform rod 6 ft. long is supported at its middle point, and weights of 8 lbs. and 6 lbs. are hung at its ends. What third weight must be hung midway between the 6-lb. weight and the fulcrum to preserve equilibrium?

5. In last question, how many inches from the fulcrum must a 20-lb. weight be hung to preserve equilibrium?

6. A rod which balances on its fulcrum when unweighted carries a 20-lb. weight at 3 ft. from the fulcrum on one side, and a 40-lb. weight at 2 ft. from the fulcrum on the other side. Which will preponderate, and what weight hung at 18 in. from the fulcrum will be sufficient to restore equilibrium?

7. Weights of 7, 8, 4, and 3 lbs. are hung at 1, 2, 3, and 4 ft. from one end of a rod whose weight may be neglected, the rod being free to turn on a hinge at this end. What force must act upwards at the other end of the rod to sustain them, the length of the rod being 6 ft.?

8. Calculate the pressure on the hinge in last question. (Take moments round the other end.)

III.—Parallel Forces and Centre of Gravity. Arts. 15–29.

1. A uniform straight lever 10 ft. long balances at a point 3 ft. from one end, when 12 lbs. are hung from this end and an unknown weight from the other. The lever itself weighs 8 lbs. Find the unknown weight.

2. A straight lever 6 ft. long weighs 10 lbs., and its centre of gravity is 4 ft. from one end. What weight at this end will support 20 lbs. at the other, when the lever is supported at 1 ft. distance from the latter?

3. Two equal weights of 10 lbs. each are hung one at each end of a straight lever 6 ft. long, which weighs 5 lbs.; and the lever, thus weighted, balances about a point 3 in. distant from the centre of its length. Find its centre of gravity.

4. A uniform lever 10 ft. long balances about a point 1 ft. from one end, when loaded at that end with 50 lbs. Find the weight of the lever.

5. A straight lever 10 ft. long, when unweighted, balances about a point 4 ft. from one end; but when loaded with 20 lbs. at this end and 4 lbs. at the other, it balances about a point 3 ft. from the end. Find the weight of the lever.

6. A lever is to be cut from a bar weighing 3 lbs. per ft. What must be its length that it may balance about a point 2 ft. from one end, when weighted at this end with 50 lbs.? (The solution of this question involves a quadratic equation.)

7. A lever is supported at its centre of gravity, which is nearer to one end than to the other. A weight P at the shorter arm is balanced by 2 lbs. at the longer; and the same weight P at the longer arm is balanced by 18 lbs. at the shorter. Find P.

8. Weights of 2, 3, 4, and 5 lbs. are hung at points distant respectively 1, 2, 3, and 4 ft. from one end of a lever whose weight may be neglected. Find the point

about which the lever thus weighted will balance. (This question and the following one are best solved by taking moments round the end of the lever. The sum of the moments of the four weights is equal to the moment of their resultant.)

9. Solve the preceding question, supposing the lever to be 5 ft. long, uniform, and weighing 2 lbs.

10. Find, in position and magnitude, the resultant of two parallel and oppositely directed forces of 10 and 12 units, their lines of action being 1 yard apart.

11. Weights of 1, 2, and 3 ounces are placed at the corners A, B, C of a triangle. Show that their centre of gravity is the middle point of a certain line from C to the side A B. Also show that their centre of gravity divides a certain line from A to B C in the ratio of 5 to 1, and that it divides a certain line from B to A C in the ratio of 2 to 1.

12. Show that if 4 equal weights are placed at the 4 corners of a triangular pyramid, their centre of gravity is the same as that of the pyramid. (See art. 27.)

IV.—Stability. Arts. 30, 31.

1. Two equal weights hanging from the two ends of a chain which passes over a fixed pulley without friction balance one another. Show that the equilibrium is neutral if the weight of the chain be left out of account, and unstable if it be taken into account.

2. Show that there are two positions of stable equilibrium for a uniform hemisphere lying on a table. Are there any positions of unstable equilibrium?

3. Show that a stick is in stable equilibrium when its two ends are pulled outwards by equal forces; and in unstable equilibrium when they are pushed inwards by equal forces.

V.—Work. Art. 35.

1. How many ft.-lbs. of work are done in raising a ton to a height of 10 ft.?

2. How many ft.-lbs. of work are done in raising 8·8 cub. ft. of water through a height of 60 ft.?

3. One horse-power being defined as the power to do 33,000 ft.-lbs. per minute, how many horse-power will be required to raise 2376 cub. ft. of water per hour from the bottom of a mine 200 ft. deep?

VI.—Mechanical Powers. Arts. 37–51.

1. On an inclined plane whose height is $\frac{1}{8}$ of its length, what power acting parallel to the plane will sustain a weight of 112 lbs. resting on the plane without friction?

2. The height, base, and length of an inclined plane are as the numbers 3, 4, 5. What weight will be sustained on the plane without friction by a power of 100 lbs. acting (*a*) parallel to the base, (*b*) parallel to the plane?

3. Find the ratio of the power applied to the pressure produced in a screw-press without friction, the power being applied at a distance of 1 ft. from the axis of the screw, and the distance between the threads being $\frac{1}{8}$ in.

4 In the system of pulleys in which one cord passes round all the pulleys, its different portions being parallel, what power will sustain a weight of 2240 lbs. without friction, if the number of cords at the lower block be 6?

VII.—Falling Bodies. Arts. 63–66.

Assuming that a falling body acquires a velocity of 32 ft. per sec. by falling for 1 sec., find:—

1. The velocity acquired in 12 sec.

2. The distance fallen in 12 sec.

3. The distance that a body must fall to acquire a velocity of 10 ft. per sec.

4. The time of rising to the highest point, when a body is thrown vertically upwards with a velocity of 160 ft. per sec.

5. The height to which a body will rise if thrown vertically upwards with a velocity of 32 ft. per sec.

6. The velocity with which a body must be thrown vertically upwards that it may rise to a height of 25 ft.

7. The velocity that a body will have after 3 sec., if thrown vertically upwards with a velocity of 100 ft. per sec.

8. The height that the body in last question will have ascended.

9. The velocity that a body will have after $1\frac{1}{2}$ sec., if thrown vertically downwards with a velocity of 30 ft. per sec.

10. The distance that the body in last question will have described.

VIII.—Atwood's Machine. Art. 67.

Two weights are connected by a cord passing over a pulley as in Atwood's machine, friction being neglected, and also the masses of the pulley and cord; find:—

1. The acceleration when one weight is double of the other.

2. The acceleration when one weight is to the other as 20 to 21.

3. The velocity acquired in 10 sec., when one weight is to the other as 39 to 41.

4. The velocity acquired in moving through 5 ft. when the weights are as 19 to 21.

5. The distance through which the same weights must move that the velocity acquired may be half that in last question.

6. The distance through which two weights which are as 49 to 51 must move that they may acquire a velocity of 2·4 ft. per sec.

IX.—Composition of Motions. Arts. 70, 71.

1. A ship sails through the water at the rate of 10 miles per hour, and a ball rolls across the deck in a direction perpendicular to the course, at the same rate. Find the velocity of the ball relative to the water.

2. The wind blows from a point intermediate between N. and E. The northerly component of its velocity is 5 miles per hour, and the easterly component is 12 miles per hour. Find the total velocity.

3. The wind is blowing due N.E. with a velocity of 10 miles an hour. Find the northerly and easterly components.

4. A stone is projected horizontally from a point 144 ft. above a level plain, with a velocity of 100 ft. per sec. How long will it take to reach the ground, and what horizontal distance will it have travelled?

5. A person who can row at 5 miles an hour has to cross a river 400 ft. wide, which runs at 3 miles an hour. Show that if he rows at right angles to the stream he will land at a point 240 ft. down the stream, and that in order to land at the point opposite where he started he must row a distance of 500 ft. through the water.

6. A stone is thrown at an elevation of 30° with velocity V. Show that the vertical component of the velocity is $\frac{1}{2}$V at the beginning, and is $\frac{1}{2}V - gt$ after t seconds. Also that the time to the highest point is $\frac{V}{2g}$ and the whole height $\frac{V^2}{8g}$.

ANSWERS.

I.—Ex. 1. 10. Ex. 2. 141·4 lbs. Ex. 3. 12 lbs. Ex. 4. 13 lbs. Ex. 5. 20 lbs. in second, 28·28 lbs. in first. Ex. 6. 40 lbs. Ex. 7. 34·6 lbs. Ex. 8. Angles of 120°. Ex. 9. Equal to either, and bisecting the contained angle. Ex. 11. 5 lbs., 13 lbs.

II.—Ex. 1 28 lbs. Ex. 2. 40 in. from smaller weight. Ex. 3. 60 lbs. by A, 40 lbs. by B. Ex. 4. 4 lbs. Ex. 5. 3·6 in. Ex. 6. The 40 lbs.; $13\frac{1}{3}$ lbs. Ex. 7. $7\frac{5}{8}$ lbs. Ex. 8. $14\frac{1}{8}$ lbs.

III.—Ex. 1 $2\frac{6}{7}$ lbs. Ex. 2. 2 lbs. Ex. 3. 15 in. from centre. Ex. 4. $12\frac{1}{2}$ lbs. Ex. 5. 32 lbs. Ex. 6. 10·4 ft. nearly.

Ex. 7. 6 lbs. Ex. 8. $2\frac{6}{7}$ ft. from end. Ex. 9. $2\frac{3}{16}$. Ex. 10. 2 units acting at 5 yds. from greater force.

V.—Ex. 1. 22,400 ft.-lbs. Ex. 2. 33,000 ft.-lbs. Ex. 3. 15 H.P.

VI.—Ex. 1. 14 lbs. Ex. 2. (*a*) $133\frac{1}{3}$ lbs.; (*b*) $166\frac{2}{3}$ lbs. Ex. 3. 1 to 603 nearly. Ex. 4. $373\frac{1}{3}$ lbs.

VII.—Ex. 1. 384 ft. per sec. Ex. 2. 2304 ft. Ex. 3. $1\frac{9}{16}$ ft. Ex. 4. 5 sec. Ex. 5. 16 ft. Ex. 6. 40 ft. per sec. Ex. 7. 4 ft. per sec. upwards. Ex. 8. 156 ft. Ex. 9. 78 ft. per sec. Ex. 10. 81 ft.

VIII.—Ex. 1. $\frac{1}{3}g$. Ex. 2. $\frac{1}{11}g$. Ex. 3. 8 ft. per sec. Ex. 4. 4 ft. per sec. Ex. 5. 1 ft. 3 in. Ex. 6. 4 ft. 6 in.

IX.—Ex. 1. 14·14 miles per hour. Ex. 2. 13 miles per hour. Ex. 3. 7·07 miles per hour each. Ex. 4. 3 sec.; 300 ft.

HYDROSTATICS.

85. We shall now treat of the laws of force as applied to fluids. This branch of the general science of dynamics is called *hydrodynamics* (ὕδωρ, water), and is divided into *hydrostatics* and *hydrokinetics*. Our discussions will be almost entirely confined to hydrostatics.

FLUIDS.—TRANSMISSION OF PRESSURE.

The name *fluid* comprehends both liquids and gases.

86. A fluid at rest cannot exert any tangential force against a surface in contact with it; its pressure at every point of such a surface is entirely normal. A slight tangential force is exerted by fluids in motion; and this fact is expressed by saying that all fluids are more or less *viscous*. An imaginary perfect fluid would be perfectly free from viscosity; its pressure against any surface would be entirely normal, whether the fluid were in motion or at rest.

87. When pressure is uniform over an area, the total amount of the pressure, divided by the area, is called the *intensity of the pressure*. A unit of intensity frequently used is the pressure of a pound per square inch. Another unit of intensity of pressure frequently employed is "an atmosphere"—that is to say, the average intensity of pressure of the atmosphere at the surface of the earth.

88. The single word "pressure" is used sometimes to denote "amount of pressure" (which can be expressed in lbs.), and sometimes "intensity of pressure" (which can be expressed in lbs. per square inch). The context usually serves to show which of these two meanings is intended.

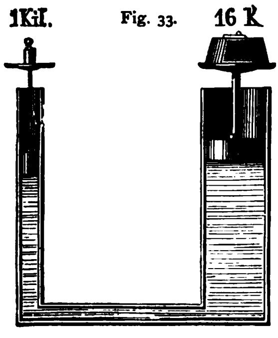
89. The intensity of pressure at any point of a fluid is the same in all directions; it is the same whether the surface which receives the pressure faces upwards, downwards, horizontally, or obliquely.

90. In this respect the action of a fluid is very different from that of a powder or mass of loose solid particles. In a heap of powder, and in an embankment of sand or any kind of earth, the pressure at a point of the interior is usually greater in the vertical than in the horizontal direction, the difference being produced by gravity and maintained by friction (or, to speak more properly, by tangential force). In a substance whose particles can slide past each other without friction no such difference can exist.

91. A heap of loose material can stand, provided its sides are not so steep that when any of the material is sprinkled upon them it slips down. This limiting gradient depends on the coefficient of friction of the material against itself; and when this coefficient is zero, as in the case of liquids, there can be no gradient at all; in other words, *the surface of a liquid at rest must be horizontal.*

92. **Principle of Transmissibility of Pressure in a Fluid.**—When a fluid is confined, if the intensity of pressure in one part be increased, by forcing in a piston, or by any other means, an equal increase will be produced in the intensity of pressure at all other parts; in other words, *pressure applied to any one part is transmitted without any change in its intensity, to all other parts.*

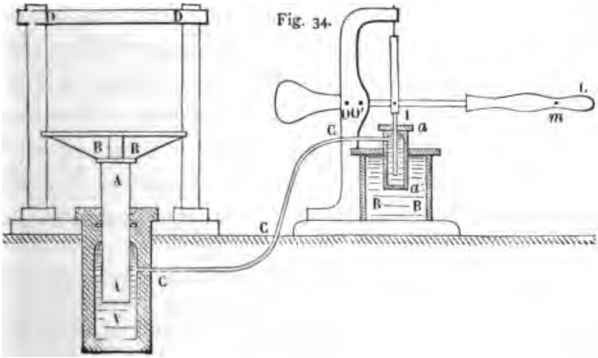
Fig. 33.



93. Hence, if there be two pistons (Fig. 33) of sectional areas A, a, working in cylinders which communicate with each other, the intervening space being filled with either

air or water, a force applied to the smaller piston to push it in will produce a greater force against the inner surface of the larger, tending to push it out. Suppose, for instance, that A is equal to sixteen times a. Then a force P applied to the smaller, produces a pressure of intensity $\frac{P}{a}$, through the whole of the inclosed fluid; and the force exerted by this transmitted pressure against the larger piston is $\frac{P}{a}$ multiplied by the area A, that is $\frac{PA}{a}$ or 16 P. Such an arrangement, therefore, constitutes a machine with a mechanical advantage $\frac{A}{a}$.

94. When the smaller piston is pushed in through a distance h, the pressure being supposed to remain constant,



the volume $a\,h$ of fluid is pushed out of the smaller cylinder and into the larger. This implies that the large piston moves outwards through a distance $\frac{ah}{A}$, that is $\frac{1}{16}\,h$. Thus the work which the machine gives out is $16\,P \times \frac{1}{16}\,h$, which is identical with P h, the work done in pushing in the smaller piston.

95. In the hydraulic press, or Bramah press, shown in section in Fig. 34, the pistons are replaced by plungers,

that is solid cylinders which enter the water. The pressure of the liquid on the *sides* of a plunger has no tendency to drive it either in or out, so that it is only necessary for our purpose to consider the pressures on the *ends*. A plunger may therefore be regarded as equivalent to a piston of the same sectional area, and it has the practical advantage of being more easily made water-tight.

96. The smaller plunger is worked by a lever handle; and the bodies to be pressed are placed above the larger plunger. During the up-stroke of the small plunger the water is prevented from returning by a valve, and a fresh supply of water enters the small cylinder through another valve, which communicates with a reservoir. In the down-stroke this latter valve closes, and the former opens. If L and l are the lengths of the longer and shorter arms of the lever, a force F exerted in pushing down the handle produces a force $F\frac{L}{l}$ pressing the small plunger downwards, and this, by the foregoing principles, produces a force $F\frac{L}{l}\cdot\frac{A}{a}$, urging the large plunger upwards. The total "mechanical advantage" of the press is therefore $\frac{L}{l}\cdot\frac{A}{a}$, or is the product of the mechanical advantage $\frac{L}{l}$ of the lever-handle by the mechanical advantage $\frac{A}{a}$, due to the inequality of the areas of the plungers.

When the *diameters* of the plungers are given, the student must bear in mind that the areas of two circles (or of any two similar figures) are as the squares of their diameters. Thus, if the diameters are as 1 to 10, the value of $\frac{A}{a}$ will be 100.

PRESSURE AS DEPENDENT ON DEPTH.

97. In a uniform liquid at rest the intensity of pressure is the *same at all points at the same depth*. For a cylindrical portion of the liquid, such as A B (Fig. 35), would be pushed from A towards B if the pressure at A were greater than that at B.

98. Again, consider a cylindrical portion with its axis vertical, as $m' m$ (Fig. 36). It is urged downwards by its

Fig. 35.

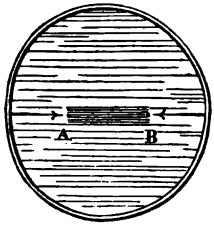
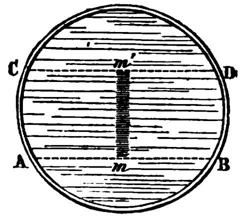


Fig. 36.



own weight and by the pressure on its upper end, and is urged upwards by the pressure on its lower end. The pressures on its sides urge it neither upwards nor downwards. Hence the amount of pressure on a horizontal area at any level AB is greater than that on an equal horizontal area at a higher level C D, by the weight of a cylinder of the liquid whose base is equal to either of these areas, and whose height is the difference of levels.

99. By taking the upper end of our cylinder in the free surface—that is, the surface exposed to the air, we obtain the following rule:—*The pressure on a horizontal area at any depth is greater than the pressure of the atmosphere on an equal area by the weight of a cylinder of the liquid whose base is equal to this area, and whose height is equal to the given depth.*

If we suppose that the surface of the liquid is free from pressure, a perfect vacuum being preserved above it, the pressure on a horizontal area at any depth will be equal to the weight of the aforesaid cylinder.

100. This weight is the product $A h w$ of three factors, namely, the area A, the depth h, and the weight w of unit volume of the liquid. If h be expressed in inches, A must be in square inches, and w will be the weight of a cubic inch of the liquid. The intensity of pressure due to the depth is found by putting $A = 1$, and is $h w$.

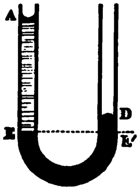
The actual intensity of pressure at any depth in a liquid is the pressure due to the depth *plus* the pressure at the surface.

101. The pressure on an area which is not horizontal is most intense on those parts which are deepest, and the average pressure over any area is equal to the actual pressure at what is called its centre of gravity; that is to say, the point which would be the centre of gravity of a thin uniform plate coinciding with the area. The total amount of pressure (in addition to the transmitted atmospheric pressure) on any area A is therefore A h w; where w denotes, as before, the weight of unit volume of the liquid, and h denotes the depth of the centre of gravity of the area.

102. The resultant pressure on a horizontal area acts at its centre of gravity, since the pressure is uniformly distributed over it; but the pressure on a vertical or oblique plane area is not uniformly distributed; it increases with the depth, and the resultant therefore acts at a point lower down than the centre of gravity. The point at which the resultant acts (that is, the point where the line of action of the resultant cuts the area) is called the *centre of pressure.* It is usual, in determining the centre of pressure, to leave atmospheric pressure out of account, and attend only to the pressure due to the weight of the liquid.

103. When the dimensions of the area are very small in comparison with the depth (as in the case of an area an inch square at the depth of a mile in the sea), the pressure is sensibly uniform over it, and the centre of pressure is therefore sensibly coincident with the centre of area. The same remark applies to the pressure of the atmosphere over an area measuring even several yards in length and breadth.

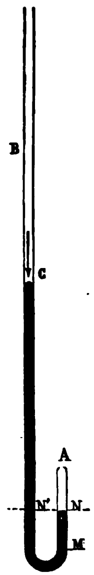
Fig. 37.



104. When two liquids which do not mix occupy the two arms of a bent tube, as in Fig. 37, where A E represents water and E D mercury, the heavier liquid

will occupy the lowest part of the tube, and the intensity of pressure at the surface of junction E will be equal to that at E′, a point at the same level in the other arm. Hence the pressure due to the depth A E of the lighter liquid must be the same as that due to the depth D E′ of the heavier. That is, the heights of the free surfaces A and D (both supposed to be at the same atmospheric pressure) above the surface of junction will be inversely as the densities of the two liquids. The density of mercury is about $13\frac{1}{2}$ times that of water; hence the height A E in the figure should be about $13\frac{1}{2}$ times D E′.

Fig. 38.



If only a single liquid occupy the tube, it will stand at the same level in both branches.

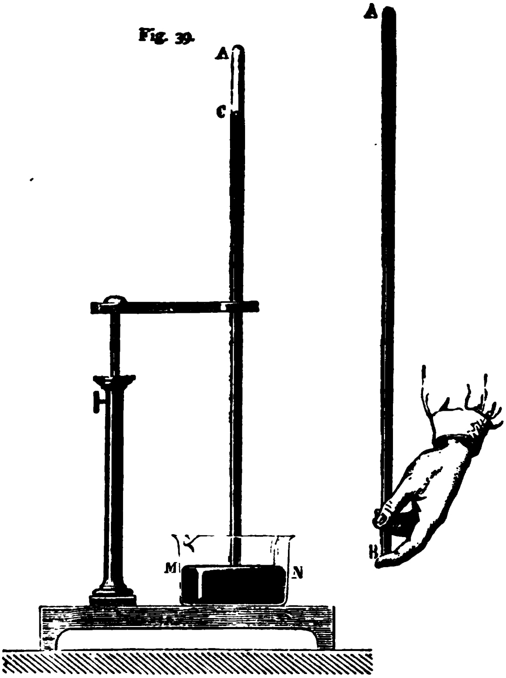
105. Thus far we have been supposing both ends of the tube to be open, and in free communication with the atmosphere. If the tube be closed at one end, as in Fig. 38, so as to imprison a portion of air A N, the level of the liquid will not be the same in both branches, except when the imprisoned air happens to be at the same pressure as the external air. If this air is at greater pressure than the external air (which will be the case if the liquid has been introduced by pouring it in at the open end without moving the tube), the pressure at N, the point of junction of the liquid with the inclosed air, will be equal to that at the same level N′ in the open branch, and will exceed atmospheric pressure by the pressure due to the depth C N′ of the liquid. In general, if h denote the difference of levels of one and the same liquid in the two branches of a bent tube, or in any two connected vessels, and w the weight of unit volume of the liquid, then $h\,w$ will be the difference of the pressures at the free surfaces. The lower of the two free surfaces is that which is exposed to the greater pressure.

106. If there is a perfect vacuum over one of the two

free surfaces, so that this surface is altogether exempt from pressure, hw will be equal to the pressure at the other free surface. This is the principle of the common barometer, in which the pressure of the air is balanced by that of a column of mercury with a vacuum above it.

107. Torricellian Experiment.—Let a glass tube

Fig. 39.



about 3 feet long, closed at one end and open at the other, be filled with mercury (Fig. 39). Then, keeping the finger pressed against the open end, let the tube be inverted, and the open end plunged in a vessel of mercury. If the finger be then removed the mercury will fall in the tube, leaving

a nearly vacuous space above it; and the top of the column of mercury in the tube will be found to be about 30 inches higher than the surface of the mercury in the cistern. This experiment was first made by Torricelli.

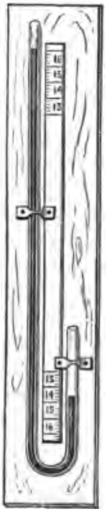
108. The weight of a cubic inch of mercury is nearly half a pound. Hence, when the barometer stands at 30 inches the pressure of the atmosphere is nearly 15 lbs. per sq. in. The actual pressure of the atmosphere at a given place varies considerably from time to time, the difference between its greatest and smallest values being about one-fifteenth of the mean pressure. Seventy-six centimetres, which is usually adopted as the standard height of the mercurial column, is about 29·92 English inches. The reason why we do not feel this pressure in ordinary circumstances is that the liquids in the pores and passages of our bodies are at an equal or greater pressure.

109. It is easy to compute the resultant force due to atmospheric pressure upon a plane surface, since we have merely to multiply the intensity of the pressure by the area of the surface. A similar computation applied to a surface which is not plane gives what is called the total pressure upon it; but this is an element of less interest than the resultant pressure. If a body has one plane face, the resultant pressure upon the remainder of its surface is easily found, for it must be equal and opposite to the pressure on the plane face, since atmospheric pressure, when acting all round a body, does not tend to move the body as a whole. The force with which the air holds the receiver of an air-pump against the plate on which it rests is equal to the area of the base of the receiver multiplied by the difference between the intensities of the pressures without and within.

110. In the best barometers the tube is vertical, and dips into mercury at its lower end, as in the Torricellian experiment. The vessel containing this mercury is called the *cistern*. Fig. 40 represents another form, called the siphon barometer, in which the mercury is contained in a bent tube with two vertical branches. The shorter

branch is open to the air, and the vacuous space is at the top of the longer branch. Whichever form of instrument be adopted, it is the difference of levels between the mercury exposed to the vacuum and the mercury exposed to the air that must be taken as "the barometric height;" that is to say, as the height of a mercurial column which exerts the same pressure as the atmosphere.

Fig. 40.



When the pressure of the atmosphere changes, the rise of the mercury in one limb is to its fall in the other in the inverse ratio of their sections; for the same volume of mercury which enters one limb leaves the other, and if the volumes of two cylinders of mercury are equal their lengths must be inversely as their bases. A similar rule applies to the cistern barometer; but instead of the whole section of the cistern we are only to reckon that portion of it which is external to the immersed tube.

111. In order that the barometric height at a given place may be truly proportional to the pressure of the air at that place, the mercury must always have the same density. Now mercury expands with heat, and is therefore less dense when warm than when cold. To avoid error from this cause a standard temperature for the mercury has by common consent been adopted, namely, 0° C., or 32° F.; and the observed barometric height is "corrected for temperature" by calculating the length of a column of mercury of the standard temperature which would exert the actual pressure.

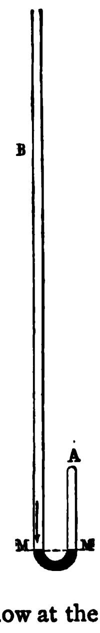
For example, if the mercury be so warm that it has only $\frac{99}{100}$ of the density of mercury at the standard temperature, the reduced height will be $\frac{99}{100}$ of the observed height. We here suppose the observed height to be known in true inches or centimetres. But, practically,

it is measured on a brass scale, which itself expands with heat and contracts with cold, so that the inches or centimetres marked upon it are only true at one particular temperature. This circumstance causes the correction for temperature to be rather smaller than it would otherwise be. If the scale expanded in length exactly as much as the mercury expands in volume, no correction for temperature would be required at all.

It is further necessary, in most barometers, to apply a small correction for capillarity (see art. 168).

LAWS OF BOYLE AND DALTON.

Fig. 41.



112. Boyle's Law (sometimes called Mariotte's law).—*When a gas is compressed, or allowed to expand, its pressure varies in the inverse ratio of its volume, provided that its temperature be kept constant.* Thus, if a quantity of air at the pressure of one atmosphere be compressed (at constant temperature) to half its original volume, it will have a pressure of two atmospheres. The same law may be thus stated; let v denote the volume and p the pressure (per unit area) of a given quantity of gas (both expressed in terms of any units we please), then the product vp will always have the same value at the same temperature. This is Boyle's law, so called from its discoverer. It may be verified by the following experiment.

113. Take a bent tube of uniform bore (Fig. 41), with both limbs vertical, the shorter being closed at the end, and the longer open. Pour in enough mercury to cut off communication between the two limbs; and when you have succeeded in making it stand at the same level M M in both branches, measure the height A M of the air in the shorter branch. This air is now at the same pressure as the external air. Next pour in mercury

till it stands about 76 centimetres, or 30 inches, higher in the open than in the closed branch; strictly speaking, this difference should be made equal to the "barometric height" at the time. It will then be found that the mercury in the closed branch has risen to a point N, half-way between M and A (Fig. 42); in other words, that the air confined in the closed limb, which is now at a pressure of two atmospheres, has only half its original volume.

Fig. 42.

114. Or take a straight tube, closed at one end and open at the other. Partially fill it with mercury, and measure the remaining portion which is occupied with air; let its length be called a. Then close the open end with the finger, and invert the tube in a cistern of mercury as in the Torricellian experiment. In this position measure the length of the inclosed air, which will be greater than before; call it b. Also measure the height of the mercury in the tube above that in the cistern; call it h, and let H denote the barometric height at the time. Then it will be found that the product Ha is equal to the product $(H-h)\,b$,—a result which verifies the law; for the expanded air is at a pressure due to a column $H-h$ of mercury, while the unexpanded air is at a pressure due to H.

115. Boyle's law is not absolutely exact. For air, oxygen, and nitrogen, the product vp is sensibly less at a pressure of twenty or thirty atmospheres than at one atmosphere; but the deviation from exactness can only be detected by measurements of great precision. For those gases which can be easily liquefied, such as carbonic acid, vp diminishes as they approach liquefaction. For hydrogen, oxygen, nitrogen, and common air (all rendered perfectly dry), at any pressures from the lowest that have been measured up to two or three atmospheres, the law is practically exact.

116. **Dalton's Law.**—When two or more gases, which do not act chemically upon one another, are mixed, with or without expansion or compression,—all measurements being made at one and the same temperature,—the sum of the separate values of vp before mixture is equal to the value of vp for the mixture. If we apply this law to the mixture of two or more portions of the same gas, it simply gives us Boyle's law, which may therefore be regarded as a particular case of Dalton's. Dalton's law may be expressed by saying that the total pressure in a mechanical mixture of gases is the sum of the pressures due to the component gases considered separately; or, *each gas exerts its own pressure just as if the others were not present.* As regards exactness, this law is about on a par with that of Boyle.

117. The effect of increase of temperature upon a gas is to increase vp. Thus, if a gas be heated at constant pressure (as when the air of a room is heated by a fire), it expands; and if it be heated at constant volume (being inclosed, for example, in an air-tight vessel), it increases in pressure. The alteration produced in vp by change of temperature is almost exactly the same for hydrogen, oxygen, nitrogen, and common air. Hence the value of vp for a gas affords a natural measure of temperature. Temperatures thus expressed are called "absolute temperatures by the air-thermometer." If T denote temperature thus reckoned, then, by definition, for any given mass of gas which undergoes changes of temperature,

$$vp \text{ varies as } T,$$

therefore $\frac{vp}{T}$ remains constant during all changes of volume, pressure, and temperature.

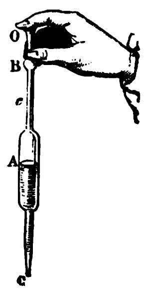
PUMPS, ETC., FOR LIQUIDS.

118. As mercury is 13½ times heavier than water, the same atmospheric pressure which sustains a column of mercury, 76 cm. or 30 inches high in a barometer, will

sustain a column of water about 10¼ metres, or 34 feet high. By making a partial vacuum at the upper end of a tube whose lower end is immersed in water exposed to atmospheric pressure, we can cause the water to rise in the tube. This is the way in which we suck up water into our mouths through a straw.

The pipette (Fig. 43) is a simple instrument often used for removing liquid from a vessel which we do not wish to disturb. It is a glass tube, open at both ends, with an enlargement in the middle. The opening at the lower end, which is inserted in the liquid to be removed, is very small. The upper end is taken in the mouth, and liquid is sucked up into the pipette, in sufficient quantity to fill the enlargement, or in any smaller quantity. The upper end is then closed either with the tongue or with a finger, and the pipette can be lifted away without its contents running out. The air which occupies the upper part of the pipette is at less than atmospheric pressure, the difference being measured by the height of the liquid column.

Fig. 43.

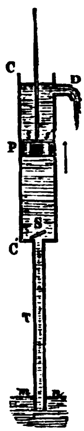


119. The filling of a syringe depends upon the same principle. As the piston is drawn up, the small quantity of air below it is expanded, and thus diminished in pressure, so that the liquid in which the lower end is immersed is forced in by the greater atmospheric pressure without.

If the space below the piston is originally full of water instead of air, the action will be still more prompt; for water does not expand like air; and if the lower end were closed, the smallest movement of the piston would suffice to produce an almost perfect vacuum. If the lower end is open, and inserted in water, the water will follow the piston (if perfectly air-tight) to any height not exceeding about 34 feet, or 10¼ metres, the precise limiting height being proportional to the height of the barometer at the time.

120. In the ordinary suction-pump, of which Fig. 44 is a section, the action, after the pump has become charged with water, is as follows. During the up-stroke of the piston P, the water below it is pushed up by atmospheric pressure acting on the surface of the water in the well *m n*, and that above it is pushed up by the piston itself till it flows out at the spout. There is a valve, or a pair of valves, *ss'*, in the piston, which can only open upwards, and these remain closed during the up-stroke, because the pressure is greater above than below them; the pressure above them, in fact, is greater, and that below them much less, than atmospheric pressure. There is another valve S at the bottom of the pump-barrel C C', which opens or closes the entrance to the descending pipe T. This valve (which, like those in the piston, can open only upwards) remains open during the up-stroke; but as soon as the piston begins to descend, it closes, and the water in the barrel, having no other way of escape, forces open the valves *ss'*. The piston is thus enabled to pass down through the water, so as to be ready to lift it out at the next up-stroke.

Fig. 44.



When the height of the spout above the barrel is considerable (it is sometimes as much as 50 feet), the pump is called a *lift-pump*.

121. The force-pump (Fig. 45) has a valve S at the bottom of its barrel, performing the same duty as the similarly placed valve in the suction-pump; and has another valve *s* opening from the bottom of the barrel into an ascending pipe R. The piston has no valve, but is solid, and, in its descent forces the water beneath it into the ascending pipe, through the valve *s*, the other valve S being closed by the pressure above it. In the

Fig. 45.

ascent of the piston, s closes, S opens, and the barrel again becomes charged with water. A plunger (see art. 95) may be used instead of a solid piston.

122. The fire-engine (Fig. 46) consists of a double-barrelled force-pump, which forces water from the two barrels alternately into a central chamber, which is air-tight at the top, and is originally full of air. A pipe, lead-

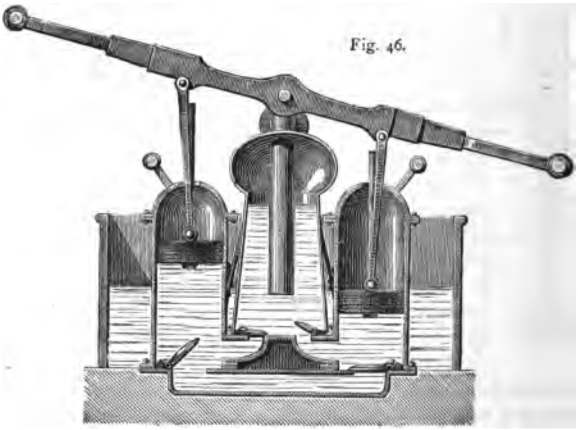


Fig. 46.

ing to the hose, opens into the lower part of this chamber, where it is constantly surrounded by water at high pressure. The air which originally filled the chamber is compressed into its upper part (hence called an air-chamber), and by its elastic reaction maintains a nearly constant rate of discharge through the hose.

123. It is not to be supposed that the employment of atmospheric pressure for the raising of water in pumps dispenses with the necessity for expending work in driving the pump. The work so expended can never be less, and is practically always more, than the work represented by the weight of water raised.

If we have a column of water 20 feet high, and 1 sq. foot

in section, sustained by atmospheric pressure, and in contact, at its upper end, with a piston which separates it from the outer air; this piston will be pressed down, on its upper side, by atmospheric pressure acting on 1 square foot of surface, that is by a force equal to the weight of about 34 cubic feet of water; while the force pressing it up from below will be equal to the weight of only $34 - 20$, that is, 14 cubic feet of water. Hence, neglecting friction and the weight of the piston, a force equal to the weight of 20 cubic feet of water will be required to sustain it.

124. During the first few strokes of a suction-pump, when the water has not yet reached the barrel, the pump is acting as an air-pump; and in many cases it does this part of its duty very imperfectly. Owing to the great expansibility of air, an air-pump is much more injuriously affected by leakage than a water-pump. Hence it is often necessary to pour in enough water to drown the valves before the pump will start.

125. The **Siphon**, is an instrument in which a liquid, under the combined action of its own weight and atmospheric pressure, flows first up-hill and then down-hill, but always in such a way as to bring about a lowering of the centre of gravity of the whole liquid mass.

In its simplest form it consists of a bent tube, one end of which is immersed in the liquid to be removed, while the other end either dips in the liquid of the receiving vessel, or discharges into the air. We shall discuss the former case, which is represented in Fig. 47. Let the surfaces $m\,n$, $m'\,n'$ of the liquid in the two vessels be at depths h, h' beneath the highest point c of the siphon (which we suppose filled with the liquid), and let each of these distances be less than the height of a column of the liquid which would balance atmospheric pressure, which

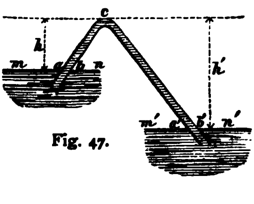


Fig. 47.

height we will call H. Then, if we suppose a diaphragm to be drawn across the siphon in any part of its length, so as to prevent flow, we shall find, on calculating the intensities of pressure on the two sides of this diaphragm, that there is an excess of pressure on the side next the vessel of higher level mn, this excess being equal to the pressure of a column of the liquid of height $h' - h$. For example, if the diaphragm is at the highest point of the siphon, the pressures on its two sides, expressed as columns of the liquid, will be respectively $H - h$ and $H - h'$; if the diaphragm is drawn across either branch, at the level of mn, the pressures on its two sides will be H and $H - (h' - h)$. If it be at the level of $m'n'$, the pressures will be $H + h' - h$ and H. Wherever we draw it we get the same difference $h' - h$. Hence, if there be no diaphragm, the liquid will be driven in the direction in which the preponderating pressure tends to urge it. As long as both ends are kept immersed, the flow will continue till the liquid attains the same level in both vessels. If the liquid in the upper vessel falls below the end of the siphon (or if the siphon is lifted out of it), air enters, and the siphon is immediately emptied of liquid. If the liquid in the lower vessel is removed, so that the discharging end of the siphon is surrounded by air, the flow will continue, unless air bubbles up the tube and breaks the liquid column. This interruption is especially liable to occur in large tubes. It cannot occur if the lower end is bent round so as to discharge the liquid in an ascending direction.

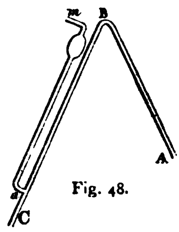


Fig. 48.

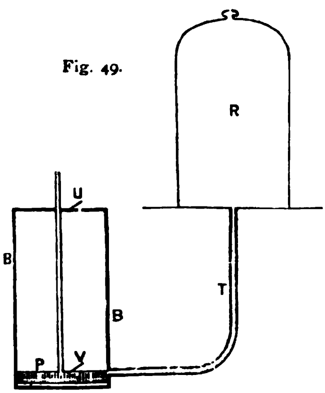
126. In order to make a siphon begin working, we must employ means to fill it with the liquid. This can sometimes be done by dipping it in the liquid, and then placing it in position while the ends are kept closed; or by inserting one end in the liquid which we wish to remove, and sucking at the other. It is usually more

convenient to apply suction by means of a side tube, as *ma* (Fig. 48), provided with an enlargement to prevent the liquid from entering the mouth. The end A of the siphon is inserted in the liquid to be removed, the other end C being kept stopped, and the operator applies suction at *m*, till the liquid flows over. In large siphons the suction is produced by a pump.

AIR-PUMPS.

127. In one of the simplest and best forms of air-pump, represented in the annexed symbolic sketch (Fig. 49), a piston P, with a valve V in it, works in a barrel B B, communicating with the outer air by a valve U at its upper end, and with the receiver R by a tube T, which enters the barrel at a small distance (equal to the thickness of the piston) from the lower end. Both valves open upwards.

Fig. 49.



In the up-stroke, the piston-valve V keeps shut, and the air above the piston is pushed out of the barrel through the valve U. In the down-stroke, U is kept closed by the preponderance of atmospheric pressure outside, and the piston-valve V opens, allowing the air to pass up through it as the piston descends to the bottom of the barrel.

128. Let R denote the joint volume of the receiver and connecting passage; B the volume of the barrel, exclusive of the space occupied by the piston and piston rod. Then, if we start with the air in the interior either at atmospheric density or at any smaller density, and the

piston at the bottom of the barrel; the effect of the first up-stroke will be to expel the contents of the barrel, and allow the air in the receiver to expand from volume R to volume R + B. The down-stroke is idle, being merely a preparation for the next up-stroke. Thus, in one complete stroke, the fraction $\frac{B}{R+B}$ of the whole contained air is expelled, while the fraction $\frac{R}{R+B}$ remains; and the density and pressure are both reduced to the fraction $\frac{R}{R+B}$ of their original values. The densities (and also the pressures) after successive strokes, thus form a geometric series whose ratio is $\frac{R}{R+B}$. For example, if R is ten times B, and we begin with the air at atmospheric density and pressure; the density and pressure after one complete stroke will be $\frac{10}{11}$, after two strokes $\frac{100}{121}$, after three strokes $\frac{1000}{1331}$, and after n strokes $(\frac{10}{11})^n$ of atmospheric density and pressure.

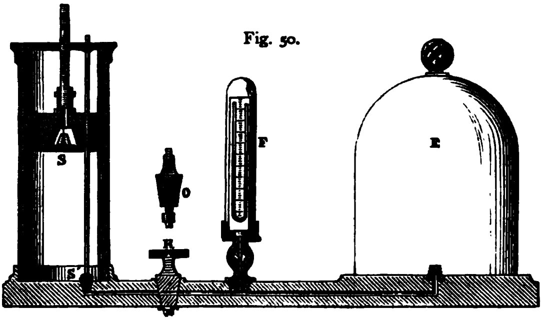
129. We have here tacitly assumed that there is no leakage; and our calculation gives what is often called the theoretical exhaustion, being the theoretical limit which actual working may approach, but cannot quite attain. A pump which is in good order will give nearly the theoretical exhaustion for several strokes at the beginning; but as the vacuum improves, the departure from the theoretical *limit of progress* becomes continually more marked. Every pump in fact has its own *limit of vacuum*, which in most cases exceeds the pressure of $\frac{1}{10}$th of an inch of mercury. Even when we succeed (as we may) in so completely preventing leakage that we can detect no change in the vacuum when the pump is left to itself, we cannot prevent the presence of a little air at atmospheric pressure between the piston and the top of the barrel at the moment of commencing the down-stroke; and during the down-stroke this air will expand and fill the barrel.

130. The valves of air-pumps are of two kinds, silk

valves and mechanical valves. In silk valves, which are the kind usually employed by English makers, there is a short and narrow slit in a thin plate of brass, and a flap of oiled silk is secured at both ends to the plate, in such a position that its central portion covers the slit. When the pressure of the air is greater on the further side of the plate than on the side where the silk is, the flap is slightly lifted and the air gets through; but excess of pressure in the opposite direction presses the flap down over the slit and makes it air-tight.

131. S and S′ in Fig. 50 are mechanical valves. The valve S′ is carried by a rod, which passes through the

Fig. 50.



piston, fitting tightly enough to be lifted by the piston when the up-stroke begins; but its ascent is almost immediately arrested by a stop near the upper end of the rod, and the piston slides upon the rod during the remainder of the up-stroke. The piston-valve S is all this time kept closed by a weak spiral spring. In the down-stroke, the piston first carries down the valve-rod with it and closes the valve S′. It then slides upon the rod till it reaches the bottom of the barrel, and during the latter part of this movement the piston-valve S is opened by the pressure of the air beneath it.

132. An air-pump is usually provided with two stop-

cocks, one for opening and intercepting communication between the barrel and the receiver; the other for admitting the external air to the latter, a precaution without which it could not be removed from the plate on which it rests. The stop-cock R in Fig. 50 (shown in section at O) does duty for both purposes, being pierced with two openings which do not communicate with each other.

133. F in the same figure is a mercurial gauge, equivalent to a siphon barometer. The difference of level of the mercury in its two branches is the measure of the pressure in the receiver. Another form of mercurial gauge is often employed, consisting of a glass tube more than 30 inches long, open at both ends. Its lower end dips in a cistern of mercury, and its upper end communicates with the receiver. The height of the mercury in the tube above that in the cistern measures the difference between the pressure in the receiver and that in the external air. Sometimes this tube is side by side with a barometer tube, both having the same cistern. Then the difference of levels in the two columns of mercury measures the difference between the vacuum in the receiver and the Torricellian vacuum in the barometer.

Fig. 51.

134. The double-barrelled pump, which is often employed, has the advantage of avoiding the idle return-stroke above-mentioned; but the same end can be attained with a single barrel by the arrangement represented in Fig. 51. In the down-stroke, the air is expelled through the piston-rod, which is hollow; and in the up-stroke,

through the spring-valve at the top of the barrel. The side passage communicates with the receiver.

135. One of the best pumps for obtaining a good vacuum is Tate's, the barrel of which is shown in section

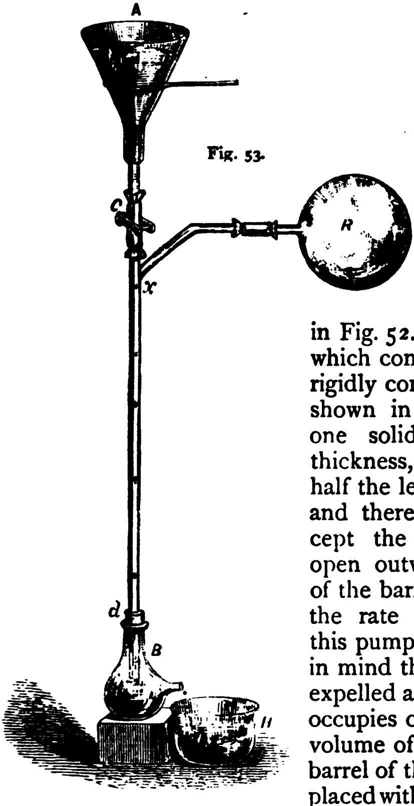
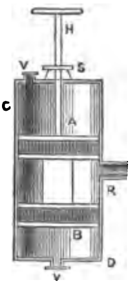


Fig. 53.

Fig. 52.



in Fig. 52. The piston A B, which consists either of two rigidly connected pistons, as shown in the figure, or of one solid piston of great thickness, occupies nearly half the length of the barrel; and there are no valves except the two, V V, which open outwards at the ends of the barrel. In computing the rate of exhaustion by this pump, it must be borne in mind that the air which is expelled at each single stroke occupies only about half the volume of the barrel. The barrel of this pump is usually placed with its axis horizontal.

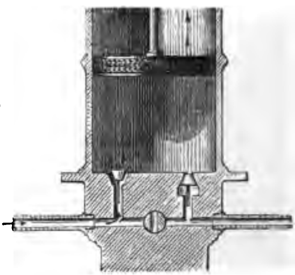
136. Sprengel's pump (Fig. 53), though too slow in its action for ordinary lecture illustration, gives a much

better vacuum than any of those above described, and has been largely used in modern physical investigations. The length xd must be more than 30 inches. Mercury from the funnel A falls in a succession of drops down the interior of the fine tube, cd, and continually sweeps before it the air which enters this tube from the receiver R. When the process has been going on for half an hour or more, the lower portion of the tube contains a continuous column of mercury sensibly equal in height to that of the barometer, and the drops fall on the top of this column with a sharp metallic clink.

COMPRESSION-PUMP.

137. All the air-pumps thus far discussed are for exhaustion. When it is desired to charge a vessel with air or gas at high pressure, a compression-pump is used. It has two valves, as shown in Fig. 54, one opening inwards and the other outwards. Air or gas is admitted into the barrel through the former (the left-hand valve in the figure), and is expelled through the latter (the right-hand valve) into the vessel which is to be charged. If the barrel is fed with air or gas at constant density, the same quantity is forced into the receiver at every stroke, and the density of the compressed air accordingly increases in arithmetical progression.

Fig. 54.



PRINCIPLE OF ARCHIMEDES.

138. A body heavier than water, such as a ball of glass or lead, requires less force to support it in water than in air or vacuum. A ball of cork will not only remain sup-

ported without help, but requires force to hold it down, when entirely immersed. In the former case the difference between the forces required to support the body in air and in vacuo, is called its *loss of weight in water*. In the latter case, the apparent weight in water is to be regarded as a negative quantity,—the body apparently gravitates upwards when submerged,—and the loss of weight in water is therefore to be computed by adding the forces required to hold it up in vacuo and to hold it down in water.

The loss of weight in water is always equal to the weight of the water whose place the body occupies when submerged. This may be established experimentally by the following excellent class illustration.

139. From one of the scales of a balance (Fig. 55) is suspended a cylindrical bucket, and below this a solid cylinder, which exactly fits it. These are balanced by weights in the other scale. A vessel of water is then placed below the bucket, in such a position that the solid cylinder will be completely immersed in it when the beam of the balance is horizontal. As long as the bucket is empty, the buoyancy of the water causes the other scale to preponderate; and on pouring water into the bucket it is found that, in order to restore the balance, the bucket must be exactly filled with water. The weight of as much water as fills the bucket is therefore equal to the loss of apparent weight in the immersed cylinder.

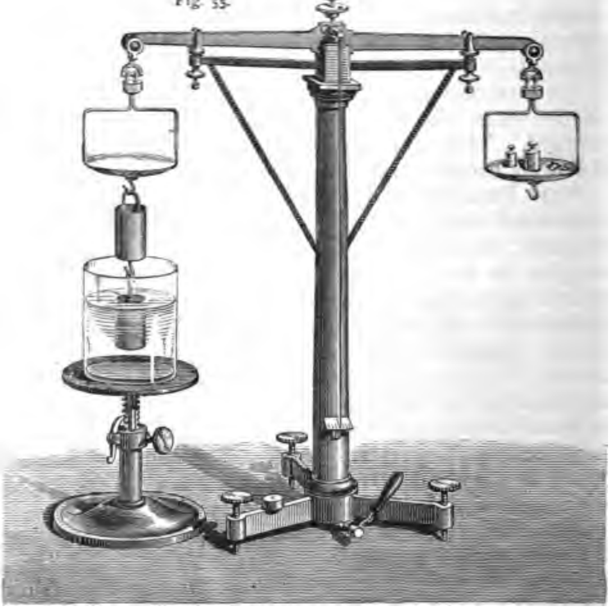
140. This result might have been predicted from the following considerations:—

When a system is in equilibrium, the equilibrium will not be destroyed by fastening any parts of it together in their actual positions. Hence the equilibrium of a liquid would not be destroyed if any portion of its interior were to become rigid without expansion or contraction. The weight of this solidified portion is therefore exactly balanced by the resultant pressure of the surrounding liquid. But this resultant pressure would be the same upon any other body occupying the same place; hence

the resultant pressure of a liquid upon any body completely immersed in it, is equal to the weight of an equal volume of the liquid, and acts vertically upwards.

Further, since the weight of the imaginary solidified portion is equivalent to a single force acting at its centre

Fig. 55.



of gravity, the resultant pressure upon a body, completely immersed in a uniform liquid, acts in a vertical line through the centre of gravity of the space occupied by the body. All this reasoning remains true whether the surface of the liquid be exposed to atmospheric pressure or not.

141. As regards a body partially immersed in a liquid,

we may divide it, in imagination, into two parts, by a horizontal plane coincident with the surface of the liquid. Let us call these two parts the immersed and the non-immersed portion. Suppose the immersed portion to be replaced by solidified liquid, and the non-immersed portion by solidified air, there will then be equilibrium. Hence the pressure on the whole surface of the body balances the weights of these imaginary solidified portions, acting at their respective centres of gravity. The loss of apparent weight as compared with weighing in vacuo, is therefore the sum of the weights of those volumes of liquid and air.

142. When a body is completely surrounded by air, similar reasoning shows that the resultant pressure upon it, or its loss of apparent weight, is equal to the weight of its own volume of air.

Neglecting the weight of this volume of air, as insignificant compared with that of the body, the resultant of the pressures over the whole surface of a body partially immersed in a liquid, is a vertically upward force through the centre of gravity of the space occupied by the immersed portion, and is equal to the weight of as much of the liquid as would fill this space.

143. Hence, still neglecting the weight of the displaced air, the conditions to be fulfilled by a body floating in equilibrium on a liquid, are—

1. That the weight of the body be equal to that of the liquid which it displaces.

2. That the centre of gravity of the body be in the same vertical with what is called the *centre of buoyancy*, that is to say, the centre of gravity of the space occupied by the immersed portion of the body.

144. The principle that a body immersed either wholly or partially in a fluid loses a portion of its weight equal to that of the fluid which it displaces, is called (from its discoverer) the *principle of Archimedes*. It applies to gases as well as to liquids, and explains the apparent upward gravitation of gases lighter than air. A cubic metre of

common air weighs about 1276 grammes, and a cubic metre of hydrogen at the same pressure and temperature weighs 88.4 grammes. Hence a cubic metre of hydrogen tends to rise through air, with a force equal to the weight of 1187.6 grammes.

145. In very accurate weighings, it is necessary to take account of the weight of the air displaced by the body weighed, on the one hand, and by the counterpoising weights on the other; unless the body happens to have the same mean density as the weights, in which case (as their volumes will be equal) the buoyancy of the air will affect them both to the same extent. If the body weighed be less dense, and therefore of larger bulk than the weights, it displaces more air than they, and the weight of this excess of air must be added to its apparent weight to obtain its true weight, which would be directly obtained by weighing with the whole balance and its contents *in vacuo.* This is what is always meant by "weighing in vacuo." "Weighing in water" commonly means that the body to be weighed is submerged, but not the balance nor the counterpoising weights.

146. The best determinations of the weight of air and other gases have been made by employing two equal glass globes, one of them being hermetically sealed, while the other is alternately filled with the gas to be examined, and exhausted, the pressure of its contents being observed in each case at the temperature 0° C. The two globes hang from the scale-pans of a delicate balance, and the difference of their weights is observed in each case. Hence the weight of the gas which fills the globe at 0° C. and at a known pressure is found by subtraction. The fact of the globes being of the same size renders it unnecessary to take account of the buoyancy of the external air, since this buoyancy affects both alike.

A cubic foot of air, with the barometer at 30 in. and the thermometer at about 50° F. or 10° C., weighs an ounce and a quarter.

DETERMINATION OF DENSITIES OF SOLIDS AND LIQUIDS.

147. By the *absolute density* (or simply the *density*) of a substance we mean the mass of unit volume of it, for example, the number of lbs. in a cubic foot of it. By *relative density* we mean the ratio of its density to that of some standard substance, or its heaviness as compared with the standard substance bulk for bulk. When water is taken as the standard substance, relative density is usually called *specific gravity;* so that the *specific gravity of a substance* means the *weight of any volume of it divided by the weight of an equal volume of water.* Absolute density is of course equal to relative density multiplied by the density of the standard substance.

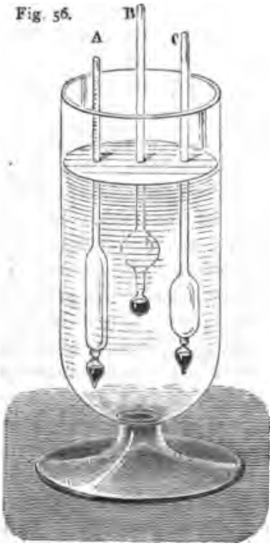
148. The most convenient British units for dealing with densities are the cubic foot and the ounce, because a cubic foot of water may be taken as weighing 1000 ozs., this being only about $1\frac{1}{4}$ oz. in excess of the truth. Hence when we express densities in ounces per cubic foot the density of water may be called 1000 and the density of any other substance will be 1000 times its specific gravity. If we express densities in lbs. per cubic foot the multiplier will be $62\frac{1}{2}$. But centimetres and grammes are much more convenient, for the weight of a cubic centimetre of cold water is one gramme to a very high degree of accuracy indeed. The density of a substance, when expressed in grammes per cubic centimetre, is therefore equal to its specific gravity.

149. The density of a substance can be directly determined by weighing a measured volume of it. This method can be easily applied to solids of rectangular form; since their volumes can be computed by multiplying together their length, breadth, and thickness. It can also be applied to liquids, by employing a vessel of known content. The vessel usually employed for this purpose is a bottle of thin glass, fitted with a *perforated stopper*, so that it can be filled and stoppered without leaving a space

for air. The difference between its weights when full and empty is the weight of the liquid which fills it; and the quotient of this by the volume occupied (which can be found by weighing the bottle when filled with water), is the density of the liquid.

150. The volume and density of a solid body of irregular shape, or consisting of a quantity of small pieces, can be determined by putting it into such a bottle, and weighing the water which it displaces. The most convenient way of doing this is to observe (1) the weight of the solid; (2) the weight of the bottle full of water; (3) the weight of the bottle when it contains the solid, together with as much water as will fill it up. If the third of these results be subtracted from the sum of the first two, the remainder will be the weight of the water displaced; which, when expressed in grammes, is the same as the volume of the body expressed in centimetres. The weight of the body, divided by that of the water displaced, is the density of the body.

Fig. 56.



151. The following methods depend upon the principle of Archimedes:—

For rough and rapid determinations of the density of liquids the *hydrometer* is usually employed. This instrument, some forms of which are exhibited in Fig. 56, consists of a closed glass vessel, whose upper part is a cylindrical tube, with graduations marked upon it. Below this is an enlargement, and, still lower, is a space containing mercury or shot, which serves,

by its weight, to keep the instrument upright when floating in a liquid. It displaces a mass of the liquid equal to its own mass, and therefore sinks deepest in the lightest liquids. The observation consists in noting the division which is at the level of the liquid surface. If D denote the density of the liquid, and V the volume displaced, V D will be the mass of liquid displaced, and this is equal to the mass of the hydrometer,—a constant quantity. Hence D varies as $\frac{1}{V}$, and V varies as $\frac{1}{D}$. If the divisions on the stem are equidistant, they correspond to values of V which are in arithmetical progression; and the corresponding values of D will be in harmonic progression. On the other hand, if the divisions are so placed as to correspond to equal differences of density, the values of V will be in harmonic progression. Both these systems of division are in use.

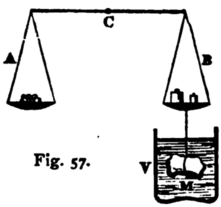


Fig. 57.

152. One of the commonest ways of determining the density of a solid body is to weigh it first in air and then in water (Fig. 57), the counterpoising weights being in air. Since the loss of weight due to its immersion in water is equal to the weight of an equal bulk of water, we have only to *divide the weight in air by this loss of weight*. Thus, from the observations

Weight in air,	125 gm.
Weight in water,	100 „
Loss of weight,	25 „

we deduce

$$\frac{125}{25} = 5 = \text{relative density.}$$

153. If the body is lighter than water we may employ a sinker,—that is, a piece of some heavy material attached to it, and heavy enough to make it sink. It is not neces-

sary to know the weight of the sinker in air, but we must observe its weight in water. Call this s. Let w be the weight of the body in air, and w' the weight of the body and sinker together in water. Then w' will be less than s. The body has an apparent upward gravitation in water equal to $s - w'$, and the weight of the water which it displaces is $w + s - w'$. Hence the relative density of the body is $\frac{w}{w+s-w'}$.

154. The densities of liquids are sometimes determined by observing the loss of weight of a solid immersed in them, and dividing by the loss of weight of the same solid in water. Thus, from the observations

Weight in air,	200 gm.
Weight in liquid,	120 ,,
Weight in water,	110 ,,

we deduce

Loss in liquid 80. Loss in water 90.

$$\text{Relative density of liquid} = \frac{80}{90} = \frac{8}{9}.$$

155. The volume of a solid body, especially if of irregular shape, can usually be determined with more accuracy by weighing in a liquid than by any other method. If it weigh w grammes in air, and w' grammes in water, its volume is $w - w'$ cubic centimetres, since it displaces $w - w'$ grammes of water. The mean diameter of a wire can be very accurately determined by an observation of this kind for volume, combined with a direct measurement of length. The volume divided by the length will be the mean sectional area, which is equal to πr^2, where r is the mean radius.

156. Density of a mixture. Let volumes v_1, v_2, v_3 of different ingredients be mixed together, without swelling or shrinking, so that the volume of the mixture is simply the sum $v_1 + v_2 + v_3$. Let the densities of the ingredients be d_1, d_2, d_3, and their masses m_1, m_2, m_3; so that we have

$$m_1 = v_1 d_1. \qquad m_2 = v_2 d_2. \qquad m_3 = v_3 d_3.$$

The mass of the mixture will be $m_1 + m_2 + m_3$; and if we divide this by its volume, we obtain its density, which is therefore

$$\frac{m_1 + m_2 + m_3}{v_1 + v_2 + v_3}. \qquad (1)$$

When the *volumes* and densities of the ingredients are given, it is convenient to throw this formula into the form

$$\frac{v_1 d_1 + v_2 d_2 + v_3 d_3}{v_1 + v_2 + v_3}. \qquad (2)$$

When the *masses* and densities of the ingredients are given, it is convenient to throw it into the form

$$\frac{m_1 + m_2 + m_3}{\frac{m_1}{d_1} + \frac{m_2}{d_2} + \frac{m_3}{d_3}} \qquad (3)$$

Similar formulæ can be employed, whatever be the number of ingredients.

Swelling or shrinking will affect the denominator in each of these expressions, but not the numerator, since the mass of the whole must always be the sum of the masses of the parts. Thus, for example, if a mixture shrinks in the ratio of 20 to 19, so that its volume is $\frac{19}{20}$ of the sum of the volumes of its constituents, its density will be $\frac{20}{19}$ of the value given by the above formulæ.

DETERMINATION OF HEIGHTS BY THE BAROMETER.

157. The pressure at any point in the interior of a liquid, due to the weight of the liquid, is wh, w denoting the weight of unit volume of the liquid, and h the height of the free surface above the given point.

Let P denote the pressure and W the weight of unit volume of the air at any point. Then, if we write

$$P = W H, \text{ or } H = \frac{P}{W},$$

H will denote the height (measured upwards from the

given point) of an imaginary atmosphere of uniform density the same as that at the point in question, which would produce the actual pressure at the point. The change of pressure as we ascend or descend *for a short distance* in the actual atmosphere is sensibly the same as it would be in this imaginary "homogeneous atmosphere;" hence if H be expressed in feet an ascent of one foot takes off $\frac{1}{H}$ of the total pressure, just as an ascent of one foot from the bottom of an ocean 60,000 feet deep takes off $\frac{1}{60,000}$ of the pressure.

158. Since W varies as P for air at constant temperature, the equation $H = \frac{P}{W}$ shows that for a given temperature H is the same at all pressures. In ascending by successive steps of 1 foot, in an atmosphere $\frac{P}{W}$ or H will be the same at all heights, and each step will take off $\frac{1}{H}$ of the current pressure. The pressures will therefore form a series in which each term will be $1 - \frac{1}{H}$ or $\frac{H-1}{H}$ of the preceding one. The pressures will therefore diminish in geometrical progression as the heights increase in arithmetical progression.

159. When the temperature is not uniform, the value of $\frac{P}{W}$ varies directly as the "absolute temperature by air-thermometer" (art. 117). Hence H varies directly as the absolute temperature. The average value of H is about 26,000 ft., or about 5 miles.

160. To determine the height of a mountain by means of the barometer, simultaneous observations, both of the pressure and temperature of the air are made at its base and summit. A mean value of H for the intervening body of air is calculated by employing the arithmetical mean of the two observed temperatures, and the rest of the calculation is made on the assumption of a geometrical rate of decrease.

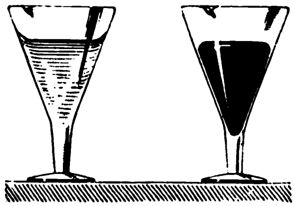
161. From the above value of H it appears that an

ascent of 260 ft. from any level diminishes the pressure by 1 per cent; for example, it would reduce the pressure from 30 in. to 29.7 in., or from 20 in. to 19.8 in.

CAPILLARITY.

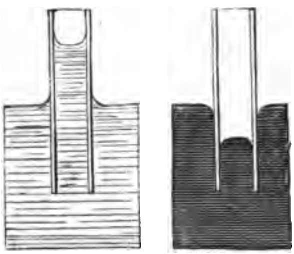
162. In the statements which we have thus far made respecting the positions assumed by liquids in equilibrium, we have neglected the effects of what is called *capillarity.* If the surface of the water in a half-filled glass vessel, whose interior has been wetted all over, be carefully observed, it will be seen not to be level near the edges, but to be bent upwards as in Fig. 58. Mercury, on the other hand, does not wet glass, and if some of this liquid be put into a glass vessel, its surface will be depressed at the edges, as shown in Fig. 59.

Fig. 58. Fig. 59.



163. When a fine tube is dipped in a liquid, the departure from a uniform level becomes still more marked. Water stands much higher within a fine glass tube than without it. Mercury, on the other hand, stands lower within than without. Figs. 60, 61, represent sections of two glass tubes dipped into water and mercury respectively. The surface of the liquid within the tube is concave in the one case, and convex in the other. The general law for all liquids, and tubes of all materials, is, that if the

Fig. 60. Fig. 61.



liquid wets the tube, it will stand higher within the tube than without, and the surface of the elevated column will be concave; on the other hand, if the liquid does not wet the tube, it will stand lower within the tube than without, and the surface of the liquid within the tube will be convex.

164. If the volume of that portion of the water within the tube which is above the general external level, be divided by the sectional area of the tube, the quotient is called the *mean elevation* of the water by the capillary action of the tube; and if the volume of the empty space between the surface of the mercury in the tube and the general external level be divided by the sectional area of the tube, the quotient is called the *mean depression* of the mercury. The mean elevation and mean depression are both found to vary precisely in the inverse ratio of the internal diameter of the tube; so that to obtain the most marked effects we must employ the smallest tubes.

165. When the free surface of a liquid is plane, the pressure at any point within it is found by adding the atmospheric pressure at the surface to the pressure due to the depth; and when the surface is partly plane and partly curved, as in Figs. 58, 59, the same principle can be applied in calculating the pressure at any point which is vertically under a plane portion of the surface. But when the surface is concave, the pressure at any point beneath it is less than this calculation would give. In the elevated column of water within a glass tube, the pressure is less than atmospheric. In fact, referring to Fig. 60, if we compare two points in the liquid at the same level, one of them within and the other without the tube, the pressures at these points will be equal, and the change of pressure as we ascend in the tube follows the usual law. There is therefore an abrupt change, from atmospheric pressure, immediately above the surface of the elevated column, to a pressure decidedly less than atmospheric, immediately below this surface. There is an equally abrupt change of pressure in passing

through the convex surface of the mercury within the tube, Fig. 61; but in this case the pressure is decidedly greater just below than just above the surface. In both cases the superficial film of the liquid behaves like a tightly stretched elastic band, which, when it is forced to assume a curved shape, exerts normal pressure against the body on its concave side. In Fig. 60 air is on the concave side, in Fig. 61 mercury.

166. Capillarity tends to depress the column of mercury in a barometer. The reading is always taken at the summit of the convexity,—in other words, at the highest point of the surface; but even this is too low, for the pressure beneath a convex surface of liquid is, as above stated, greater than the pressure due to the depth. Hence the correction for capillarity in a barometer is to be *added* to the observed reading. It is barely sensible in half-inch tubes; but in tubes of an eighth of an inch in diameter it is so large as to cause great trouble and uncertainty.

For a fuller treatment of the subject of capillarity we may refer to our edition of *Deschanel*, pages 124–136.

EXAMPLES IN HYDROSTATICS.

X.—Bramah Press. Arts. 92–96.

1. The sectional area of the small plunger in a Bramah press is 1 sq. in., and that of the larger 100 sq. in. The lever handle gives a mechanical advantage of 6. What weight will the large plunger sustain when 1 cwt. is hung from the handle?

2. The diameter of the small plunger is half an inch; that of the larger 1 foot. The arms of the lever handle are 3 in. and 2 ft. Find the total mechanical advantage.

XI.—Intensity of Pressure due to Depth. Arts. 97–100.

Find, in grammes per sq. cm., atmospheric pressure being neglected:—

1. The pressure at the depth of a kilometre in sea water of density 1.025.

2. The pressure at the depth of 65 cm. in mercury of density 13.59.

3. The pressure at the depth of 2 cm. in mercury of density 13.59, surmounted by 3 cm. of water of unit density, and this again by 1½ cm. of oil of density .9.

In the three following examples a cubic foot of water may be assumed to weigh 1000 oz. or 62½ lbs. Atmospheric pressure is to be neglected:—

4. Find in lbs. per sq. ft. the pressure at the depth of 100 ft. in water.

5. Find in lbs. per sq. in. the pressure at the depth of 34 ft. in water.

6 Find in lbs. per sq. in. the pressure at the depth of 30 in in mercury of sp. gr. 13.6.

Find, in inches of mercury of density 13.6, atmospheric pressure being included, and the barometer being supposed to stand at 30 inches:—

7. The pressure at the depth of 10 ft. in water.

8. The pressure at the depth of 340 ft. in water.

9. At what depth, in brine of sp. gr. 1.1, is the pressure the same as at a depth of 33 feet in water?

10. At what depth, in oil of sp. gr. .9, is the pressure the same as at the depth of 10 inches in mercury of sp. gr. 13.596?

XII.—Amount of Pressure on Immersed Areas. Arts. 99–101.

Find, in grammes weight, the amount of pressure (atmospheric pressure being neglected):—

1. On a triangular area of 9 sq. cm. immersed in naphtha of sp. gr. .848; the centre of gravity of the triangle being at the depth of 6 cm.

2. On a rectangular area 12 cm. long, and 9 cm. broad, immersed in mercury of sp. gr. 13.596; its highest and lowest corners being at depths of 3 cm. and 7 cm. respectively.

3. On a circular area of 10 cm. radius, immersed in alcohol of sp. gr. .791, the centre of the circle being at the depth of 4 cm.

4. On a triangle whose base is 5 cm. and altitude 6 cm., the base being at the uniform depth of 9 cm., and the vertex at the depth of 7 cm., in water.

5. On a sphere of radius r centimetres, completely immersed in a liquid of sp. gr. s; the centre of the sphere being at the depth of h centimetres. [The amount of pressure in this case is not the resultant pressure.]

XIII.—Examples on the Barometer, not involving Boyle's Law. Arts. 107–111.

1. A bent tube, having one end open and the other closed, contains mercury, which stands 10 in. higher in

the open than in the closed branch. Compare the pressure of the air in the closed branch with that of the external air, the barometer at the time standing at 30 in.

2. The cross sections of the open and closed branches of a siphon barometer are as 6 to 1. What distance will the mercury move in the closed branch, when a normal barometer alters its reading by 1 inch?

3. Show that, in a siphon barometer of uniform bore, the rise and fall of the highest point of the mercurial column are only half the rise and fall of the theoretical barometer.

4. If the section of the closed limb of a siphon barometer is to that of the open limb as a to b, show that a rise of 1 cm. in the mercury in the closed limb corresponds to a rise of $\frac{a+b}{b}$ cm. of the theoretical barometer.

5. Show that the same result holds for the cistern barometer, if a denote the section of the tube, and b the section of the cistern exclusive of the space occupied by the tube and its contents.

[Some barometers have their scales "compensated," the distances marked as inches or centimetres being $\frac{b}{a+b}$ of true inches or centimetres. Others, again, are furnished with a screw at the bottom, which either moves the scale or alters the level of the mercury in the cistern; and the observer turns this screw until the zero point of the scale coincides with the surface of the mercury. In a few barometers the height is read off both in the tube and in the cistern, and the difference taken.]

6. The volumes of a given quantity of mercury at 0° C. and 100° C. are as 1 to 1.0182. Compute the height of a column of mercury at 100°, which will produce the same pressure as 30 in. of mercury at 0°.

7. The volumes of a given mass of mercury, at 0° and 20°, are as 1 to 1.0036. Find the height reduced to 0°, when the actual height, at a temperature of 20°, is 30.2 in.

XIV.—Laws of Boyle and Dalton. Arts. 112–116.

1. In performing the Torricellian experiment a little air is left above the mercury. If this air expands a thousand-fold, what difference will it make in the height of the column of mercury sustained when a normal barometer reads 30 in.?

2. In performing the Torricellian experiment, an inch in length of the tube is occupied with air at atmospheric pressure, before the tube is inverted. After the inversion, this air expands till it occupies 15 inches, while a column of mercury 28 inches high is sustained below it. Find the true barometric height.

3. The mercury stands at the same level in the open and in the closed branch of a bent tube of uniform section, when the air confined at the closed end is at the pressure of 30 inches of mercury, which is the same as the pressure of the external air. Express, in atmospheres, the pressure which, acting on the surface of the mercury in the open branch, compresses the confined air to half its original volume, and at the same time maintains a difference of 5 inches in the levels of the two mercurial columns.

4. At what pressure (expressed in atmospheres) will common air have the same density which hydrogen has at one atmosphere; their densities when compared at the same pressure being as 1276 to 88.4?

5. Two volumes of oxygen, of density .00141, are mixed with three of nitrogen, of density .00124. Find the density of the mixture—(*a*) if it occupies five volumes; (*b*) if it is reduced to four volumes.

XV.—Pumps and Siphons. Arts. 118–126.

1. Find the force required to sustain the piston of a suction-pump without friction, if the radius of the piston be 3 in., the depth from it to the surface of the water in the well 12 ft., and the height of the column of water

above it 2 ft. Show that the answer does not depend on the size of the pipe which leads down to the well.

2. Two vessels of water are connected by a siphon. A certain point P in its interior is 10 cm. and 30 cm. respectively above the levels of the liquid in the two vessels. The pressure of the atmosphere is 1000 grammes weight per sq. cm. Find the pressure which will exist at P—(*a*) if the end which dips in the upper vessel be plugged; (*b*) if the end which dips in the lower vessel be plugged.

XVI.—Air-pump Arts. 127–137.

1. If the receiver has double the volume of the barrel, find the density of the air remaining after 10 strokes, neglecting leakage, &c.

2. Air is forced into a vessel by a compression pump whose barrel has $\frac{1}{10}$th of the volume of the vessel. Compute the density of the air in the vessel after 20 strokes.

3. In the pump of Fig. 49 show that the excess of the pressure on the upper above that on the lower side of the piston, at the end of the first up-stroke, is $\frac{B}{R+B}$ of an atmosphere [in the notation of art. 128]; and hence that the first stroke is more laborious with a small than with a large receiver.

4. In Tate's pump show that the pressure to be overcome in the first stroke is nearly equal to an atmosphere during the greater part of the stroke; and that when half the air has been expelled from the receiver, the pressure to be overcome varies, in different parts of the stroke, from half an atmosphere to an atmosphere.

XVII.—Density and Specific Gravity. Arts. 147–154.

1. A rectangular block of stone measures 3 ft. × 1½ ft. × ¾ ft. and weighs 540 lbs. Find its density in ounces per cubic foot, and its specific gravity.

2. A specific gravity bottle holds 500 grains of water, and 900 grains of sulphuric acid. Find the sp. gr. of the acid.

3. A certain volume of mercury of sp. gr. 13.6 weighs 100 grains, and the same volume of another liquid weighs 14.8 grains. Find the sp. gr. of this liquid.

4. Find the mean section of a tube 16 cm. long, which holds 1 gm. of mercury of sp. gr. 13.6.

5 A bottle filled with water weighs 2120 grains. Five hundred grains of filings are thrown in, and the water which flows over is removed, still leaving the bottle just filled. The bottle then weighs 2540 grains. Find the sp. gr. of the filings.

6. Find the sp. gr. of a body which weighs 58 oz. in air, and 46 oz. in water.

7. Find the sp. gr. of a body which weighs 63 oz. in air, and 35 oz. in a liquid of sp. gr. .85.

8. A glass ball loses 330 grains when weighed in water, and 390 grains when weighed in a saline solution. Find the sp. gr. of the solution.

9. A body, lighter than water, weighs 102 oz. in air; and when it is immersed in water by the aid of a sinker, the joint weight is 23 oz. The sinker alone weighs 50 oz. in water. Find the sp. gr of the body.

XVIII.—Relation between Volume and Loss of Weight in a Liquid. Art. 155.

All volumes to be stated in cubic centimetres, and all weights in grammes.

1. Find the volume of a solid which weighs 357 gm. in air, and 253 gm. in water of unit density.

2. Find the volume of a solid which weighs 458 gm. in air, and 409 gm. in brine of density 1.2.

3. How much weight will a body whose volume is 47 cubic cm. lose, by weighing in a liquid whose density is 2.5?

4. Find the weights in air, in water, and in mercury, of a cubic cm. of gold of density 19.3.

5. A wire 1293 cm. long loses 508 gm. by weighing in water. Find its mean section.

6 A copper wire 2156 cm. long weighs 158 gm. in air, and 140 gm. in water. Find its volume, specific gravity, and mean section.

7 What will be the weights, in air and in water, of an iron wire 1000 cm. long and a millimetre in diameter, its density being 7.7?

XIX.—Floatation.

1 What volume of water will be displaced by 12 cub. ft. of oak, of sp. gr. .9, floating on it?

2 What volume of mercury, of sp. gr. 13.6, will be displaced by 5 cub. in. of iron, of sp. gr. 7.5, floating on it?

XX.—Density of Mixtures. Art. 156.

The density of water is to be taken as unity.

1. Find the mean density of a combination of 8 parts by volume of a substance of density 7, with 19 of a substance of density 3.

2. Find the mean density of a combination of 8 parts by weight of a substance of density 7, with 19 of a substance of density 3.

3. What volume of fir, of density .5, must be joined to 3 cub. in. of iron, of density 7.1, that the mean density of the whole may be unity?

4. What mass of fir, of density .5, must be joined to 3 lbs. of iron, of density 7.1, that the mean density of the whole may be unity?

5. Two parts by volume of a liquid of density .8, are mixed with 7 of water, and the mixture shrinks in the ratio of 21 to 20. Find its density.

6. A mixture of gold, of density 19.3, with silver, of

density 10.5, has the density 18. Assuming that the volume of the alloy is the sum of the volumes of its components, find how many parts of gold it contains for one of silver—(*a*) by volume; (*b*) by weight.

ANSWERS.

X.—Ex. 1. 30 tons. Ex. 2. 4608.

XI.—Ex. 1. 102500. Ex. 2. 883.35. Ex. 3. 31.53. Ex. 4 6250. Ex. 5. 14.76. Ex. 6. 14.76. Ex. 7. 38.8. Ex. 8 330 in. or $27\frac{1}{2}$ ft. Ex. 9. 30 ft. Ex. 10. 12 ft. 7 in.

XII.—Ex. 1. 45.79. Ex. 2. 7342. Ex. 3. 994. Ex. 4. 125. Ex. 5. $4 \pi r^2 h s$.

XIII.—Ex. 1. $1\frac{1}{3}$ atm. Ex. 2. $\frac{8}{7}$ in. Ex. 6. 30.546 in. Ex. 7. 30.09 in.

XIV.—Ex. 1. .03 in. Ex. 2. 30 in. Ex. 3. $2\frac{1}{6}$. Ex. 4. .0693. Ex. 5. (*a*) .001308, (*b*) .001635.

XV.—Ex. 1. 687 lbs. Ex. 2. (*a*) 970, (*b*) 990 gm. per. sq. cm.

XVI.—Ex. 1. $\frac{1}{68}$ of an atm. nearly. Ex. 2. 3 atm.

XVII.—Ex. 1. 2560, 2 56. Ex. 2. 1.8. Ex. 3. 2.013. Ex. 4. .0046 sq. cm. Ex. 5. 6.25. Ex. 6. $4\frac{5}{6}$. Ex. 7. 1 9125. Ex. 8. $1\frac{2}{11}$. Ex. 9. $\frac{102}{128}$.

XVIII.—Ex. 1. 104. Ex. 2. $40.8\dot{3}$. Ex. 3. 117.5. Ex. 4. 19.3, 18.3, 5.7. Ex. 5. .393 sq. cm. Ex. 6. 18 c.c., 8.7, .00835 sq. cm. Ex. 7. 60.48, 52.62.

XIX.—Ex. 1. 10.8 cub. ft. Ex. 2. 2.76 cub. in.

XX.—Ex. 1. 4.185. Ex. 2. 3.6115. Ex. 3. 36.6 cub. in. Ex. 4. 2.577 lbs. Ex. 5. 1.0033. Ex. 6. (*a*) 5.77, (*b*) 10.6.

HEAT.

TEMPERATURE AND THERMOMETERS.

167. Two bodies are said to have the *same temperature* if neither of them gains heat from the other when they are put in contact.

On touching a substance which has a lower temperature than the hand we receive a sensation of cold, because the hand loses heat; on touching a substance which has a higher temperature than the hand we receive a sensation of warmth, because the hand gains heat from the substance.

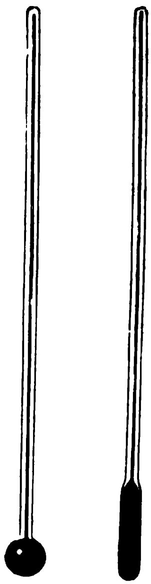


Fig. 62.—Thermometers.

Transference of heat also occurs between bodies of different temperatures when placed opposite to one another without direct contact.

A body of lower temperature will continue to gain heat from a body of higher temperature until the difference of temperature between them is reduced to nothing.

168. Temperature is commonly measured by means of the expansion produced by gain of heat, and the corresponding contraction produced by loss of heat. The mercurial thermometer is the instrument commonly employed for this purpose. It consists of a fine tube of glass, enlarged at one end into a bulb (Fig. 62), and containing mercury enough to fill the bulb

and part of the tube. The tube is hermetically sealed at the end remote from the bulb, and the portion which is void of mercury is usually vacuous; the vacuum having been obtained by boiling the mercury and sealing the tube while full of mercurial vapour.

169. If a thermometer at the temperature of cold water is plunged into warm water, both the glass and the mercury expand. If they expanded equally, the end of the mercurial column would stand always at the same point of the tube; but, in point of fact, the mercury expands much more than the glass, and the end of the mercurial column accordingly advances along the tube.

If a thermometer is to be so constructed that the end of the column shall move over a large distance for even a small change of temperature, it must have a large bulb and a fine tube. Doubling the capacity of the bulb would have the same effect in this respect as halving the sectional area of the tube.

170. In order to render the indications of different thermometers comparable, two definite temperatures have, by common consent, been selected as standards of reference. One of these is the temperature of a mixture of snow and water, or of pounded ice and water. This temperature is briefly styled the *freezing point*. The other is the temperature of the steam of boiling water under mean atmospheric pressure, and is called the *boiling point*.

171. The imaginary standard thermometer has a perfectly uniform tube, which is divided into equal parts by divisions etched upon it.

In the Centigrade scale, which is regarded as the standard scale of temperature for scientific purposes, the interval between the freezing and the boiling point is divided into 100 equal parts, numbered in order from 0 at freezing point to 100 at boiling point. These parts are called degrees Centigrade, and the number read off at the end of the mercurial column is called the temperature in degrees Centigrade.

172. As actual tubes are never perfectly uniform in bore, they are *calibrated* whenever special accuracy is required. That is to say, a small column of mercury is passed along the tube before joining it to the bulb, and marks are made on the tube showing the length occupied by this column in different parts of it. The information thus obtained enables the constructor, after the thermometer has been closed, to divide the space between freezing and boiling point, or between any two fixed points, into parts of equal capacity; and this is what theory requires.

173. If the scale of a thermometer extends below freezing or above boiling point, the degrees beyond these limits should be of the same capacity as those between them. When the scale does not extend so far as these limits, two fixed points must be determined by comparison with another thermometer.

174. Alcohol or some other spirit is often employed instead of mercury. Spirit expands about five times as much as mercury; nevertheless the bulbs of spirit thermometers are usually made larger than those of mercurial thermometers, because the tube must be of large bore to prevent the column of spirit from becoming broken into parts, with bubbles between them. Owing partly to the large size of the bulb, and partly to the thermal qualities of the liquid, spirit thermometers are, as a rule, much slower in their action than mercurial thermometers, that is to say, they occupy a much longer time in coming to the temperature of the surrounding medium.

175. If a spirit thermometer is constructed so as to agree with a standard mercurial thermometer at all temperatures, the degrees of the spirit thermometer will not be parts of equal volume, but will increase in volume in proceeding from the bottom to the top of the scale. In other words, spirit is more expansible at high than at low temperatures, when judged by comparison with mercury. Spirit thermometers are, in fact, always constructed upon this plan, the mercurial thermometer being regarded as the standard instrument.

176. If a mercurial thermometer reads correctly when first constructed, it will read too high after the lapse of a few months, unless a long time has been allowed to elapse between closing and graduating it. This change is probably owing, at least in part, to the gradual yielding of the thin glass which forms the bulb, under the preponderating external pressure. It affects the readings at all temperatures alike, and is commonly denoted by the name *change of zero.*[1]

There is very little change of zero in spirit thermometers. The great expansibility of spirit renders any slight change in the glass of little consequence. Glass expands by about $\frac{1}{38700}$ of its volume for each degree Centigrade, mercury by about $\frac{1}{5550}$, and alcohol by about $\frac{1}{1000}$.

177. Besides the Centigrade scale above described, there are two other thermometric scales in extensive use. Réaumur's scale, which is very much used on the continent of Europe (being older than the Centigrade scale), differs from it only in the length of its degrees, four of which are equal to five degrees Centigrade. Boiling point is accordingly marked 80° on this scale.

Fahrenheit's scale, which is older than either of the others, is the favourite scale for popular purposes in this country. It differs from the other two in the length of its degrees and also in its zero-point. As regards the length of its degrees, 9 of them are equal to 5 of the Centigrade scale, or to 4 of Réaumur's. As regards its zero, this is 32 of its own degrees below the freezing point. The interval from freezing to boiling point consists of 180 degrees, and as the former is marked 32° the latter is marked 212°. Fig. 63 will assist in conveying a clear idea of the relations between the three scales. The abbreviations, Cent., Reau., Fahr., or still more frequently

[1] "Change of zero" in any graduated instrument means that the whole scale is bodily shifted, so that distances are measured from a different starting-point.

the initial letters C., R., F., are used to denote the particular scale employed.

178. In questions of reduction from one of these scales to another, if one of the two be the Fahrenheit scale, the student should in the first place consider whether the number which is to be reduced denotes an interval, or a temperature.

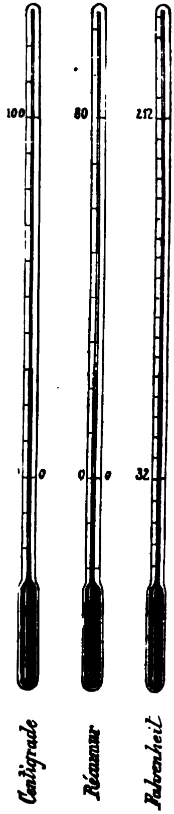


Fig. 63.—Scales.

To reduce an *interval* of temperature from the Cent. to the Fahr. scale we must multiply by $\frac{9}{5}$. For example, an interval of 10° C. is an interval of 18° F.

To reduce a *temperature* from the Cent. to the Fahr. scale we must first multiply by $\frac{9}{5}$ and then add 32. For example, the temperature 10° C. is the temperature $(18° + 32° =)$ 50° Fahrenheit.

To reduce an *interval* of temperature from the Fahr. to the Cent. scale we must multiply by $\frac{5}{9}$.

To reduce a *temperature* from F. to C. we must first subtract 32 and then multiply by $\frac{5}{9}$. For example, the temperature 62° F. is $(30° \times \frac{5}{9} =)$ $16°\frac{2}{3}$ Centigrade.

Temperatures below zero, in any scale, are marked with the *minus* sign. Thus a temperature ten degrees lower than zero is marked $-10°$. The following are examples of reducing temperatures below zero:—

(1.) Reduce the temperature $-20°$C. to the F. scale.

$$-20 \times \frac{9}{5} = -36.$$
$$-36 + 32 = -4. \quad \text{Answer, } -4° \text{ F.}$$

(2.) Reduce the temperature 14° F. to the C. scale.

$$14 - 32 = -18.$$
$$-18 \times \tfrac{5}{9} = -10. \quad \text{Answer, } -10^\circ \text{ C.}$$

In all countries the Centigrade scale is regarded as the standard scale for scientific reference, and we shall always employ it hereafter, unless we give warning to the contrary.

EXPANSION.

179. Nearly all substances expand when heated. In the case of solids we have sometimes to speak of change of volume and sometimes of change of length.

If we conceive a solid to increase its length by 1 per cent., while its breadth and thickness remain unaltered, its volume will increase by 1 per cent. If we then conceive its breadth to increase by 1 per cent. the volume will receive a fresh addition of 1 per cent. If we lastly conceive its thickness to increase by 1 per cent., the volume will again be increased by 1 per cent.; making a total increase of about 3 per cent. in volume for an increase of 1 per cent. in each of the three linear dimensions. Hence an expansion of $\frac{1}{100}$ linear taking place equally in all directions, is an expansion of about $\frac{3}{100}$ cubical; and in general when expansion is equal in all directions, the cubical expansion (or expansion of volume) is three times the linear expansion.

180. The expansion (whether linear or cubical) due to a change of 1° in temperature is called the *coefficient of expansion*, and may be very different for the same substance at different temperatures. It usually increases as the temperature increases, the temperature being measured by a mercurial thermometer graduated in the manner described in art. 158.

The mean coefficient of expansion between 0° and 100° is computed by dividing the total expansion between

these limits by 100. Thus, if a substance whose length at 0° is denoted by 1 has the length 1·02 at 100°, its total expansion from 0° to 100° is ·02, and its mean coefficient of expansion between 0° and 100° is ·0002.

There may sometimes be a question as to whether "the coefficient of expansion at a stated temperature" signifies

$$\frac{\text{increase of length for one degree}}{\text{length } \textit{at that temperature}},$$

or signifies

$$\frac{\text{increase of length for one degree}}{\text{length } \textit{at zero}}.$$

The difference between the two values is usually insignificant in questions relating to solids. When the difference is material, as it is in the case of gases, the latter of the two values is adopted; that is to say, the dimensions at zero are made the unit of comparison.

181. When the linear expansions in three mutually perpendicular directions are not equal, the expansion of volume is their sum.

The increase of capacity of a thermometer or other hollow vessel of uniform material is the same as the increase in volume of a piece of the material which would initially fill the hollow. It is therefore an example of cubical expansion.

182. The following is a table of linear expansions:—

SUBSTANCE.	Length at 100° C. of a rod whose length at 0° is 1.
Glass,	1·000729 to 1·000918
Platinum,	1·000857 to 1·000884
Iron, wrought,	1·001156 to 1·001235
,, cast,	1·001072 to 1·001109
Steel,	1·001079 to 1·001240
Gold,	1·001230 to 1·001552
Copper,	1·001712 to 1·001722
Brass,	1·001855 to 1·001895
Silver,	1·001909 to 1·001951
Lead,	1·002788 to 1·002848
Zinc,	1·002976

183. The following is one of the best modes of observing the linear expansion of a solid:—

A bar of the substance is immersed in a mixture of ice and water till it has had time to acquire the temperature of the mixture, which is 0° C. Its length at this temperature is measured by means of a metallic measuring bar immersed in a similar mixture, and furnished with travelling microscopes fitted for the measurement of very small distances. The bar whose expansion is to be observed is then immersed in a bath of boiling water, and when it has had time to come to the temperature of this water its length is again measured by means of the same measuring bar, which is kept all the time immersed in the mixture at 0° C.

It is obvious that if the measuring bar were raised to the same temperature as the bar which it is employed to measure, the observation would give relative and not absolute expansion.

184. The most direct mode of measuring the expansion of a liquid is based upon the hydrostatical principle that, if two liquid columns produce equal pressures, their heights are inversely as their densities.

If two vertical columns of the same liquid, at different temperatures, communicate with each other by means of a fine horizontal tube connecting their lower ends, their surfaces will be at different levels, and the heights of these surfaces above the axis of the connecting tube will be inversely as the densities, or directly as the volumes occupied by equal masses.

Thus, if one column is surrounded by a mixture of ice and water, and the other is surrounded by water which is kept at the temperature 10° C., and if the heights of the surfaces above the axis of the connecting tube are as 1 to 1·002, the inference is that the expansion of volume of the liquid from 0° to 10° C. is ·002. The absolute expansion of mercury has been very carefully determined by this method.

185. The relative expansion of mercury with respect to glass can be determined by means of a mercurial thermometer whose bulb and stem have been gauged before

introducing the mercury. It is necessary to know how many degrees of the stem are equal to the capacity of the bulb. We can thus express, in degrees of the stem, the total volume of the mercury at any temperature within the range of the instrument. This is a merely relative determination, because a degree of the stem is not a constant volume, but increases with the temperature.

186. If the mercury occupies n degrees at 0° and $(1 + a)\, n$ degrees at t°, a is the apparent or relative expansion from 0° to t°.

If the volume of a degree is d when the temperature is 0°, and is $(1 + g)\, d$ when the temperature is t°, g is the expansion of the glass.

The volume of the mercury is $n\, d$ at 0°, and at t° is $(1 + a)\, n\, (1 + g)\, d$. But if m be the real expansion of the mercury, $(1 + m)\, n\, d$ is the volume of the mercury at t°. Equating these two expressions, we have

$$1 + m = (1 + a)\,(1 + g),$$

an equation which expresses accurately the relation between the apparent expansion and the two real expansions.

187. The apparent expansion of mercury in glass can be more easily observed by means of the *weight-thermometer* (Fig. 64), which consists of a large bulb with a bent neck terminating in a fine opening. It is first completely filled with mercury at 0°, the instrument itself being also at 0°; and is then raised to the higher temperature t°. The mercury which runs out is caught and weighed. Its weight divided by that of the mercury which remains behind is the relative or apparent expansion, above denoted by the symbol a.

Fig. 64. Weight-thermometer.

188. The equation obtained in art. 173 gives, by multiplying out, and then subtracting 1 from both sides,

$$m = a + g + a g.$$

But a and g are so small that their product $a\,g$ can usually be neglected in comparison with their sum. We have therefore, practically,

$$m = a + g, \text{ and } a = m - g,$$

or the apparent expansion is the difference of the two real expansions.

189. Some of the best determinations of the expansion of glass have been made by observing the real and apparent expansions of mercury, and taking their difference.

The expansions of other liquids have usually been determined by observing their apparent expansions relative to glass, and adding the expansion of glass.

190. The following are a few examples of the expansion of liquids from 0° to 1° C.:—

Mercury,	·000179
Alcohol,	·001049
Ether,	·001513
Sulphuret of carbon,	·001140

191. The behaviour of water is peculiar. As its temperature is raised from 0° to 4° C. it contracts, and from 4° C. upwards it expands. In the neighbourhood of 4° C. its changes of volume are very slow, the expansion from 4° to 5° being only .0000082. For this reason the density of water at the temperature of maximum density is adopted as the standard of reference in stating the specific gravities (or relative densities) of other substances. The total expansion of water from 4° to 100° C. is ·04315.

192. Gases (at constant pressure) expand much more than solids or liquids; and all gases, except when near the condition of liquefaction, expand equally. If V_0 denote the volume at 0° C., the volume at t° C. is $V_0\,(1 + \alpha\,t)$, where α (called the coefficient of expansion) is ·00366, or $\frac{1}{273}$. The volume at 100° C. is therefore 1·366 V_0.

193. If we construct a thermometer in which temperatures are indicated by the expansion of air at constant

pressure, and graduate it with degrees of equal volume in such a manner that it marks 0° and 100° when at the temperatures of the freezing and boiling point respectively, its indications will agree almost precisely with those of a mercurial thermometer at all intermediate temperatures. At higher temperatures a divergency would appear; the mercury thermometer would read higher than the air thermometer, the difference increasing with the temperature. At such temperatures the air thermometer is regarded as the standard instrument. It is scarcely worth while to distinguish between the real and the apparent expansion of air in an air-thermometer; since the expansion of glass is practically insensible in comparison with that of air; but, strictly speaking, it is the absolute expansion of air that is regarded as the standard measure of temperature.

194. The volume at temperature $t°$ of a mass of air or gas which has the volume V_0 at 0°, is, as stated in last article, $V_0\,(1 + \frac{t}{273})$. If we multiply this expression by 273, it becomes $V_0\,(273 + t)$.

If, then, we shift the zero of our scale 273 degrees below the freezing point, so that the freezing point shall be marked 273, and the boiling point 373, the volume V of a given quantity of gas at given pressure will be proportional to its temperature as indicated on this new scale. This scale is called the "absolute scale of temperature by the air thermometer." Its zero is that purely imaginary temperature at which the gas would shrink to a mathematical point.

If $T°$ denote the absolute temperature equivalent to the ordinary Centigrade temperature $t°$, we have

$$T = t + 273,$$

and V varies as T when the pressure is constant.

195. If the pressure varies, let it be denoted by P; then $\frac{V\,P}{T}$ will remain constant under all circumstances.

Example. Find the volume, at 100° C. and a pres-

sure of 80 cm. of mercury, of a quantity of gas which has a volume of 1 litre at 0° and 70 cm.

Denoting the required volume by V, we have

$$\frac{V \times 80}{373} = \frac{1 \times 70}{273}; \text{ whence}$$

$$V = \frac{373 \times 7}{273 \times 8} = 1 \cdot 2 \text{ litre nearly.}$$

THERMAL CAPACITY.

196. Water is taken as the standard substance in measuring quantities of heat. The unit of heat is usually defined as the quantity of heat which must be added to unit mass of cold water to raise its temperature one degree. In discussions relating to conduction, it is more common to employ as the unit the quantity which would raise the temperature of unit *volume* of cold water one degree. When the gramme is the unit of mass, and the cubic centimetre the unit of volume, the two definitions are practically identical; and the unit of heat which they define is called the *gramme-degree.*

197. The quantity of heat required to raise the temperature of a given body one degree is called the *thermal capacity* of the body. Hence the thermal capacity of unit mass of cold water is unity; and the number which denotes the thermal capacity of a body expresses the mass of water which has the same thermal capacity as the body.

198. The thermal capacity of unit mass of a substance is called the *specific heat* of the substance. It is identical with the ratio of the thermal capacity of any mass of the substance to that of an equal mass of water. Thus, when we say that the specific heat of a substance is ·5, we mean that its thermal capacity is half that of an equal mass of water.

The specific heat of a substance will accordingly be

denoted by the same number, whatever unit of mass, and whatever scale of temperature be employed.

199. The quantity of heat required to raise unit mass of water 1° C. is not rigorously the same at all temperatures, but increases slightly with the temperature. The quantity required to raise it from 0° to 100° is about 100½ times the quantity required to raise it from 0° to 1°. When excessive accuracy is required, the definition of the unit of heat must state the precise temperature at which the water is to be taken; but it is not often that thermal measurements can be depended on within one per cent.

200. Knowing the thermal capacities of two bodies, we can calculate the common temperature to which they will come when allowed to give and take heat freely between themselves, while protected from thermal communication with other bodies.

Thus, let a ball of iron of thermal capacity i, and at the temperature 100°, be immersed in water of thermal capacity w and at the temperature 0°. If we denote the resulting temperature by $x°$, we have a body of capacity i brought down from temperature 100 to temperature x. This implies a loss of $i\,(100-x)$ units of heat. On the other hand we have a body of capacity w raised from temperature 0 to temperature x. This implies a gain of $w\,x$ units of heat. But the one body gains what the other loses, as there is supposed to be no external communication. Hence we have the equation

$$w\,x = i\,(100 - x)$$

whence we find

$$x = \frac{100\,i}{w+i}.$$

If we had supposed the water to be initially at 2°, its gain of heat would have been $w\,(x-2)$, and the final result would have been

$$x = \frac{100\,i + 2\,w}{i+w}.$$

201. When the mass and specific heat of a body are

given, its thermal capacity is found by multiplying them together. Hence we can solve such questions as the following:—

A copper ball weighing 120 gm. is raised to the temperature of 90°, and is then immersed in 1000 gm. of water at 10°. Find the resulting temperature, the specific heat of copper being ·092.

Here the thermal capacity of the ball is $120 \times \cdot092 = 11\cdot04$. The thermal capacity of the water is 1000. The resulting temperature is $\frac{11\cdot04 \times 90 + 1000 \times 10}{11\cdot04 + 1000} = \frac{10994}{1011} = 10°\cdot87$ nearly.

202. Conversely, from observation of the resulting temperature, we may compute the specific heat of the ball.

Example 1. A metallic ball, whose specific heat is to be determined, is heated to 100°, and then immersed in water at 0°. The mass of the ball is 150 gm., that of the water 1200 gm., and the resulting temperature is 1°. Compute the specific heat of the ball.

Calling it x, we have $150\,x$ as the capacity of the ball. The capacity of the water is 1200, and we have $150\,x \times 99 = 1200 \times 1$.

$$x = \frac{1200}{14850} = \cdot0808.$$

Example 2. A copper ball, of mass 100 gm. and specific heat ·09215, is heated to 100° and plunged in 800 gm. of a certain liquid at 5°. The resulting temperature is 6°·5. Find the specific heat of the liquid.

Calling it x, we have $800\,x$ as the capacity of the liquid, while that of the ball is 9·215. Hence we have $800\,x \times 1\cdot5 = 9\cdot215 \times 93\cdot5$.

$$1200\,x = 861\cdot6,\ x = \cdot718.$$

203. In accurate experiments for the determination of specific heat by the method here described, special precautions must be taken to make the loss of heat to the air and the containing vessel as small as possible, and the amount of such loss (or gain) must be allowed for in the final calculation.

204. The specific heats of nearly all solids and liquids increase with temperature. By the mean specific heat of a substance between two temperatures $t_1°$ and $t_2°$ is meant $\frac{1}{t_2 - t_1}$ of the heat required to raise unit mass of the substance from $t_1°$ to $t_2°$.

205. The following is a table of mean specific heats between 0° and 100° C.:—

Glass,	·1770	Antimony,	·0507
Iron,	·1098	Platinum,	·0335
Copper,	·0949	Mercury,	·0330
Zinc,	·0927	Water,	1·005
Silver,	·0557		

The following is a list of specific heats of liquids at atmospheric temperatures:—

Water,	1·000	Ether,	·529
Alcohol,	·548	Sulphuret of carbon,	·235

206. If we define the specific heat of a gas to be the quantity of heat required to raise unit mass of it one degree when it is not allowed to expand, but is kept at *constant volume*, the specific heat of perfectly dry air will be represented by the number ·1691 at all temperatures and pressures. If, on the other hand, we define it, as in the case of liquids and solids, to be the quantity of heat required to raise unit mass one degree when kept at *constant pressure*, the specific heat of air will be ·2375 at all temperatures and pressures. It is therefore important to distinguish between the *specific heat of a gas at constant volume* and its *specific heat at constant pressure.* The difference between the two is ·0684 for air, and for any other gas which has the same coefficient of expansion as air, it is the quotient of ·0684 by the relative density of the gas, as compared with dry air at the same pressure and temperature. The ratio of the two specific heats is 1·404 for dry air, and is sensibly the same for other gases except those which can be easily condensed into liquids.

207. The following laws are approximately true for both

specific heats in the case of all gases except those which can be easily liquefied:—

(1.) The specific heat of a given gas is the same at all pressures and at all temperatures.

(2.) The specific heats of different gases are inversely as their densities, these latter being compared at the same pressure and temperature. In other words, the thermal capacities of equal volumes of different gases are equal at the same temperature and pressure.

208. It is believed that the specific heats of all elementary gases and vapours whatever would be found to obey this second law, if they could be examined at sufficiently high temperatures. This belief is supported by the fact (known as Dulong and Petit's law) that what are called in chemistry the "atomic weights" of the elements are, with a few exceptions, approximately in the inverse ratio of their specific heats; taken in conjunction with the fact that the atomic weights of gases are directly as their densities (at the same pressure and temperature).

THERMAL PHENOMENA CONNECTED WITH SOLIDIFICATION AND EVAPORATION.

HEAT CONSUMED OR DEVELOPED IN CHANGE OF STATE.

209. A mixture of snow and water, or pounded ice and water, has a nearly uniform temperature throughout. This temperature (if the mixture be under ordinary atmospheric pressure) is 0° C.

Ice in contact with water cannot be raised above this temperature, and water in contact with ice cannot be depressed below it. It is the one temperature at which water and ice can remain in contact without either the ice melting or the water freezing. When hot water is mixed with ice at a temperature below 0° C., and the mixture is protected against external gain or loss of heat, it will (at least if agitated) attain finally a uniform temperature throughout, and one of three results will be obtained;

either (1), the whole will be water at a temperature not less than 0°; or (2), the whole will be ice at a temperature not higher than 0°; or (3), part will be water and part ice, and both will have the temperature 0°.

210. When a mixture of ice and water is continually gaining heat from without, and is at the same time continually agitated, some of the water will be at a temperature a little higher than 0°; but, unless the pieces of ice be large, and the external source of heat very active, it will be scarcely possible to detect this elevation by a thermometer plunged in the mixture. The heat which is gained from without serves to melt the ice, changing it from ice at 0° into water at 0°.

If a similar mixture is exposed to intense cold, so as to lose heat externally, the effect of this loss of heat will be to convert some of the water into ice, but if the whole be so well stirred as to prevent any sensible difference of temperature from existing between different parts, a thermometer plunged in the mixture will still indicate the temperature 0°.

211. The heat required to melt unit mass of ice without change of temperature is called the *heat of liquefaction of ice*, or the *latent heat of water*, and amounts to nearly 80 thermal units of the Centigrade scale; in other words, it would be sufficient to raise nearly 80 times its mass of water one degree Centigrade. The same quantity is given out without fall of temperature when unit mass of water freezes.

212. The following examples illustrate this subject:—

Example 1. Equal masses of ice at 0° and hot water are mixed, and the result is water at 0°. Compute the temperature of the hot water.

We may assume the mass of each to be unity, as this assumption cannot affect the answer. Then the ice requires 80 units of heat to melt it, and this heat is furnished by unit mass of hot water, which, by losing it, is reduced to the temperature 0°. Hence the initial temperature of the water was 80°.

Example 2. Ice at 0° is mixed with ten times its mass of water at 20°. Find the result.

The heat required to melt the ice (whose mass may be taken as unity) would be 80, and the heat which the water would lose if brought to 0° is 200. Hence the ice will all be melted, and the temperature of the resulting water will be above 0°, call it x°. Then 80 heat-units are consumed in melting the ice, and x units are employed in raising its temperature after melting. On the other hand, the original water parts with $10\,(20-x)$ units. Hence we have the equation

$$80+x=10\,(20-x),$$

whence we find $11\,x=120$, $x=11$ nearly; that is, the result will be water at 11°.

Example 3. Find the result when ice at 0° is mixed with 5 times its mass of water at 6°.

Here the heat which the water would lose in coming to 0° is 30, whereas 80 would be required to melt the ice. Hence the ice will not all be melted. In fact, $\frac{30}{80}$ or $\frac{3}{8}$ of it will be melted, while the other $\frac{5}{8}$ will remain ice. The temperature of the whole will be 0°.

Example 4. A pound of wet snow mixed with 5 pounds of water at 20° yields 6 pounds of water at 10°. Find the proportions of ice and water in the wet snow.

Taking a pound as the unit of mass, let x denote the mass of ice, and therefore $1-x$ the mass of water in the wet snow.

The part x consumes $80\,x$ units of heat in melting, and gains $10\,x$ units in rising in temperature.

The part $1-x$ gains $10\,(1-x)$ in rising in temperature.

The 5 pounds of water lose 50 units in falling in temperature. Hence we have

$$80\,x+10\,x+10\,(1-x)=50,$$
$$80\,x=40,\ x=\tfrac{1}{2},\ 1-x=\tfrac{1}{2}.$$

Hence the wet snow consisted of equal parts of water and ice.

213. A similar disappearance of heat occurs when water is converted into steam. When the conversion is effected at ordinary atmospheric pressure, the heat which disappears is 536 times as much as would raise the temperature of an equal mass of cold water one degree Centigrade. This is expressed by saying that the *heat of vaporization of water* or the *latent heat of steam* at atmospheric pressure is 536. The same quantity of heat reappears when the steam is liquefied at atmospheric pressure. Thus a pound of steam at 100°, if condensed in 536 pounds of cold water, would raise the temperature of this large quantity of water by 1° Centigrade.

Example. A gramme of steam at 100° C. is condensed in 10 grammes of water at 0°. Find the resulting temperature.

Calling the required temperature x, we have 536 units of heat given out by the steam in virtue of its liquefaction, and $100 - x$ units in virtue of its fall of temperature. On the other hand, we have $10\,x$ units gained by the 10 grammes of cold water. Thus we have

$$536 + 100 - x = 10\,x,$$
$$11\,x = 636,\ x = 58 \text{ nearly};$$

that is, the resulting temperature is about 58° C.

214. The following are the heats of liquefaction and of vaporization of various substances, in units of the Centigrade scale.

	Heat of Liquefaction.
Water,	79·25
Nitre,	62·975
Zinc,	28·13
Tin,	14·252
Lead,	5·369
Mercury,	2·83

	Heat of Vaporization at one Atmosphere.
Water,	535·9
Wood spirit,	263·7
Alcohol,	202·4
Ether,	90·45
Sulphuret of carbon,	86·67

It will be observed that in both lists water stands at the head of the scale. Water also stands first in the list

of specific heats (art. 205). The properties of water are therefore such as eminently tend to prevent sudden changes of temperature.

Most substances expand in melting and contract in solidifying. Some, however, including iron and ice, expand in solidifying and contract in melting. The volume of ice is to that of the water from which it is formed as 12 to 11 nearly.

HEAT AS A FORM OF ENERGY.

215. It is a familiar fact that friction produces heat. It is also a familiar fact that, in consequence of friction, more work must be spent in driving a machine than the machine will yield. These two facts stand in close connection. In the majority of cases the excess of the work done in driving a machine above the work yielded is represented by the heat arising from the friction; and in every case it is represented by energy, in some shape or other, which is generated by the action of the machine in addition to its regular work. Heat is not a substance, for we can create it in any quantity—but is a form of energy. When it is produced some other kind of energy must perish in its production; and conversely, other forms of energy can be produced by operations in which heat disappears.

216. The first accurate determinations of the quantitative relations between heat and other forms of energy were made by Joule. He arrived at the conclusion (in 1850) that the heat which raises one pound of water through 1° Fahr. is equivalent to 772 foot-pounds of mechanical energy. The best experiments which have been made since (by Joule and others) show that this result is very close to the truth.

To reduce to the Centigrade scale we must multiply by $\frac{9}{5}$, giving 1390. This is the number of foot-pounds of energy that are equivalent to one pound-degree of heat, or is the number of foot-grammes equivalent to one gramme-degree. If we introduce the metre instead of the

foot the number will be 426, since 1390 ft. are equal to 426 metres. The number (whether 772, 1390, or 426) which expresses how many units of work are equivalent to one unit of heat, is called *Joule's equivalent.*

217. It is easy to contrive arrangements in which mechanical energy shall be wholly converted into heat, but it is not possible to contrive arrangements in which heat shall be *wholly* converted into mechanical energy. In every steam-engine there is a partial conversion of heat into mechanical energy; mechanical work is done, and an equivalent quantity of heat supplied to the engine perishes in the doing of it; but there is a practical limit to the proportion of the whole heat which can be thus utilized. The remainder of the heat (and this is by far the larger portion) is given out from the engine, some of it being communicated to the air and surrounding bodies by radiation and conduction, and a still larger portion of it being carried off in the waste steam of a non-condensing engine, or in the waste hot water of a condensing engine. The student cannot too carefully attend to the fact that the total amount of heat which thus escapes from the engine is less than the heat supplied to it. The engine cannot yield work without sacrificing a portion of its heat. The heat sacrificed is the measure of the work yielded, and it is the aim both of the constructors and the users of engines to destroy as much heat as possible in proportion to the whole heat supplied. The ratio of the heat destroyed to the heat supplied is called the *efficiency of the steam*, and the largest value that it has attained in any steam-engine hitherto constructed does not exceed $\frac{1}{8}$.

218. Efficiency is promoted by a high temperature of boiler and a low temperature of condenser; for the limit of efficiency to which a theoretically perfect heat-engine of any kind would attain (as shown in more advanced treatises), is

$$\frac{T_1 - T_2}{T_1} \text{ or } \frac{t_1 - t_2}{273 + t_1}.$$

T_1 and T_2 denoting the absolute temperatures of boiler and condenser, and t_1 and t_2 the same temperatures expressed in the ordinary Centigrade scale.

219. A few examples at this stage will be useful.

Example 1. A leaden bullet, moving with a velocity of 1200 ft. per second, strikes a solid iron target and is thus brought to rest. Find the heat generated.

Let m denote the weight of the bullet expressed as a fraction of a pound. The energy of its motion computed by the rule given in art. 81 is

$$m \times (150)^2 = 22{,}500 \text{ ft. lbs.}$$

which divided by 1390 gives 16·2 pound-degrees as the equivalent amount of heat. These are pound-water-degrees.

In the circumstances supposed, iron being much harder than lead, the energy of the motion will be mainly expended in flattening the bullet, which is thus heated by internal friction of its parts. If we suppose all the heat generated to be taken up by the bullet, we can calculate the effect upon it from the following data:—

Heat of liquefaction of lead, ...	5·37
Specific heat of solid lead, ...	·0314
Temperature of fusion,.	326°
Initial temperature, say........	10°

To raise unit mass of lead from 10° to 326° requires $316 \times {\cdot}0314 = 9{\cdot}92$ units of heat

To melt it without change of temperature requires.................. 5·37

Making a total of...................... 15·29 units of heat required to raise unit mass of lead from 10° to its melting point and to melt it. As 15·29 m is less than 16·2 m, the bullet will be melted.

Example 2. The height of a waterfall is 100 feet, and the water runs away at the bottom with the same velocity with which it arrived at the top. Compute the rise of temperature due to the shock.

Here each pound of water loses 100 foot-pounds of energy of position, which is first converted into energy of motion during the descent, and then by collision converted into heat. The rise of temperature will be $\frac{100}{772}$ of a degree Fahr., or about $\frac{1}{14}$ of a degree Cent.

Example 3. Mercury is poured from a height of 100 feet into a vessel suitably placed to receive it, the arrangements being such that the liquid cannot splash over. Find its rise of temperature, supposing the heat generated to be all taken up by the mercury.

Here the heat gained by a pound of the liquid is the same as in the preceding example; but the rise of temperature will not be the same, since the specific heat of mercury is not 1 but $\frac{1}{30}$. The mercury will therefore be raised in temperature $\frac{30}{14}$, or $2\frac{1}{7}$ degrees Cent.

220. When air expands without any communication or abstraction of heat by surrounding bodies, its temperature falls, and if we know the work done in the expansion we can compute the fall of temperature. The heat lost is the equivalent of the work done, and the air falls in temperature to the same extent as if this heat were abstracted at constant volume. In like manner when air is compressed without entrance or escape of heat, its temperature is raised, and the rise of temperature is the same as if a quantity of heat equivalent to the work done in compressing it were communicated to the air at constant volume.

221. We can now understand why the specific heat of a gas is greater at constant pressure than at constant volume. When the gas is heated at constant pressure, it pushes back the surrounding air by its expansion, and thus does work against external resistance. When it is heated at constant volume it does no external work. If we compute the work done in the former case we find it to be almost precisely equal to the difference between the amounts of heat required in the two cases.

HEAT OF COMBINATION.

222. **Combustion.**—The most familiar mode of generating heat is by combustion. A gramme of carbon, in uniting with the oxygen of the air to form carbonic acid, generates about 8000 gramme-degrees of heat. A gramme of hydrogen, in uniting with oxygen to form steam, generates about 34,000 gramme-degrees. The heat of a fire is due to the union of the oxygen of the air with the carbon and hydrogen of the fuel. Fuel is a reservoir of potential energy, inasmuch as its elements are ready, whenever opportunity is given, to unite with the oxygen of the air and develop this large amount of heat.

223. The words "whenever opportunity is given" require some explanation. Fuel does not unite with oxygen unless it is first raised to a high temperature. But if we raise a small part of it to this temperature by external communication of heat, this part will, by its combustion, generate much more heat than was given it, and under proper management the ignition of this part may bring about the ignition of the whole.

Some mechanical illustrations may help us to a partial understanding of what goes on in such cases. If we have a large stone, lying near the edge of a precipice 1000 feet deep, the stone will not move over of itself; but it is ready to fall when opportunity is given, and a trifling expenditure of work in moving the stone to the edge will enable it to descend to the foot with terrific violence. This is an example of *giving opportunity for the running down of potential energy.*

224. Again, it is a common amusement with children to set a row of bricks on end, with small distances between them, so that when the first is tumbled over against the second, the whole row of bricks will fall in succession, each one knocking down the next. This is an example of the way in which the *running down of energy may be passed on from particle to particle through a large mass.* The firing of a train of gunpowder is another instance in

which the running down of energy is passed on from particle to particle.

225. In the firing of a gun there is a combination of illustrations of the kind of action we are considering. First, a little work spent in pulling the trigger releases a strong spring in the lock and brings about a smart blow with the hammer. This blow liberates the explosive energy of the percussion cap, which in its turn fires the powder. Thus we have a series of processes in which the running down of a smaller quantity of energy gives opportunity for the running down of a larger.

226. Combustion and explosion are examples of the general principle that chemical combination produces heat. It is necessary, however, to remark, by way of caution, that chemical combination is often accompanied by changes of state,—as from the solid to the liquid, or from the liquid to the solid form,—or by contraction of volume. These changes in some cases increase the heating effect due to the mere satisfying of chemical affinity; in other cases they diminish it, or even overpower it and produce cold, as in the case of what are called "freezing mixtures." One of the commonest of these is a mixture of snow and common salt. They unite to form liquid brine at the definite temperature $-22°$ C. The temperature of this mixture was intended by Fahrenheit to be the zero of his scale, but it is really $-7°\cdot6$ on the Fahrenheit scale as now defined.

227. Animal Heat.—The high temperature of the bodies of animals is maintained by the continual combination of oxygen derived from the air with substances containing carbon and hydrogen, which are derived from the animal's food. The blood receives a supply of oxygen in the lungs, and distributes it to all parts of the body. The oxidation of animal substances, with its attendant generation of heat, is thus continually going on, not in the lungs only, but wherever the blood circulates; and muscular exertion increases the quantity of matter oxidized in the muscles employed.

CONDUCTION OF HEAT.

228. When heat is applied to one end of an iron bar, the whole bar is gradually raised in temperature by heat transmitted along the substance of the bar. Heat transmitted in this manner is said to be *conducted.*

If the bar could be completely protected against the escape of heat from its surface, this process would go on till the rest of the bar attained the same temperature as the part to which heat is directly applied. In ordinary circumstances the process goes on till the escape of heat from each portion of the surface is as rapid as the supply from within. The bar is then said to have attained the *permanent condition;* or the *flow of heat* is said to be *steady.*

229. A *good conductor* is a substance which transmits heat rapidly by conduction. Such a substance is said to have high *conductivity.*

To obtain a measure of conductivity, consider any *isothermal section* of the substance, that is, any section which has the same temperature in all parts. Let this temperature be T°. Also consider another isothermal section very near the first, and let its temperature be $T + t$. The two sections will have sensibly the same area, and we shall suppose the distance between them to be sensibly the same over an area of 1 cm. Call this distance x. Then the quantity of heat transmitted in one second through this square centimetre will be directly as the difference of temperature t, and inversely as the thickness x. It may, therefore, be expressed by the formula $k\frac{t}{x}$, where k is the proper measure of the conductivity of the substance.

Our reason for selecting two sections very near together is, to insure that the quantity of heat gained or lost by the intervening lamina of the substance shall be insignificant in comparison with the whole heat transmitted through it.

230. If we wish to base our definition upon the transmission of heat through a portion not excessively thin, we

must suppose steady flow to have been attained, and must suppose the isothermal sections to be parallel planes. Then the rate of transmission of heat will be the same at all these sections, and the quantity which traverses one square centimetre of any one of them in one second will be correctly expressed by the formula above employed, $k\frac{t}{x}$, if we make t denote the difference of temperature between two isothermal sections, whose distance is x, and k the conductivity of the substance.

231. The following is an approximate list of the conductivities of a variety of substances, the units referred to being the centimetre, the gramme, and the second:—

	Conductivity.
Copper,	1·1
Iron,	·16
Lead,	·02
Ice,	·006
Marble,	·002
Glass,	·0005
Indian-rubber,	·000089
Fir (across grain),	·000088
Cotton wool (compressed),	·000034

These values are for ordinary atmospheric temperatures. The conductivity of metals decreases as their temperature increases.

232. The following examples will illustrate the use of the foregoing table:—

Example 1. How much heat is conducted in one second through an iron plate of length 20 cm., breadth 12 cm., and thickness ·5 cm., when its faces are steadily maintained at temperatures differing by 10° C.?

Answer, $\frac{\cdot 16 \times 20 \times 12 \times 10}{\cdot 5} = 768$ gm.-deg.

Example 2. How much heat is conducted per annum through each sq. cm. of horizontal area of a stratum of marble, if the temperature increases with the depth at the rate of 1° C. for 3000 cm.?

Answer, $\cdot 006 \times \frac{1}{3000} \times 60 \times 60 \times 24 \times 365 = 63$ gm.-deg.

233. Diffusivity.—When we are discussing the propagation of heat and the elevation of temperature in bodies in which steady flow of heat has not been attained, it is necessary to take into account not only conductivity but also thermal capacity. High thermal capacity is a hindrance to elevation of temperature; and if we denote the thermal capacity of unit volume of a substance by c, conductivity being denoted by k, it can be shown that a large class of phenomena depend on the value of $\frac{k}{c}$, and not upon k or c separately. This quantity $\frac{k}{c}$ is called by Sir Wm. Thomson the *diffusivity* of a substance, on account of the close analogy which it bears to the *coefficient of diffusion*, which expresses the tendency of one substance (for example, salt) to diffuse itself through another (for example, water).

RADIATION OF HEAT.

234. When I hold my hand before a bright fire, the heat which my hand receives is not conducted to it through the air. The temperature indicated by a thermometer held before the fire is not the temperature of the air, but is much higher. Any body so held acquires a temperature higher than that of the air around it, and accordingly gives heat to this air instead of receiving heat from it.

In like manner, the heat which the sun sends to the earth traverses on its way the upper regions of the atmosphere, where very intense cold always prevails.

A lens of ice will collect the sun's rays and inflame substances at the focus, in the same way as an ordinary burning-glass, though the temperature of the ice itself can never exceed zero.

235. The mode of transmission of heat of which these are instances, is called *radiation*.

The laws of radiation of heat are identical with the laws of light, and there is no essential difference between light

and radiant heat. All kinds of luminous rays are heating, but some rays which are heating do not affect the human eye with the sensation of light.

236. The emission and absorption of heat by bodies in contact with the air are due to two distinct actions, namely, radiation and *surface-conduction*,—actions which sometimes assist and sometimes oppose each other. Surface-conduction, properly so called, is due to the contact of the air with the body, and takes place more rapidly in wind than in calm air,—more rapidly in dense than in rare air. Radiation, on the contrary, is independent of the contact of the air, and goes on with at least equal activity *in vacuo*.

237. Radiation is very much affected by the nature of the surface of the radiator. The worst radiators are brightly polished metallic surfaces, and the best radiator known is carbon, which, in experiments on radiation, is generally employed in the form of lamp-black.

238. A good radiator both emits heat and absorbs heat copiously by radiation, the one power being in all cases a precise measure of the other. The blackness of charcoal is due to its complete absorption of luminous rays. The brightness of gas and candle flames is believed to be due to copious emission of luminous rays by the small particles of solid carbon which these flames contain. A flame of pure hydrogen, though hotter, is far less luminous.

239. The absorbing power of a surface is usually different for different kinds of rays. Black surfaces far exceed white ones in their power of absorbing heat from the sun and from other brightly luminous sources of heat. This difference, however, does not extend to their respective powers of absorbing dark heat, such as that emitted from a kettle filled with boiling water. White-lead, for example, is one of the best absorbers of dark heat.

Still more remarkable instances of selective absorption are exhibited by gases, and are always accompanied by a corresponding selective emission. We shall return to this

subject and discuss it more fully under the head of Optics, in connection with the spectroscope.

240. Radiant heat, while on its way from the emitting to the absorbing body, must be regarded as consisting in an affection, not of the intervening air or other gross matter, but of the luminiferous ether which permeates all matter and all space.

241. Bodies which are transparent to radiant heat are called *diathermanous*. One of the most diathermanous bodies known is rock-salt. Glass and water are far less transparent to dark than to luminous radiation, and Professor Tyndall has found a liquid (solution of iodine in sulphuret of carbon) which is eminently diathermanous to dark heat though opaque to light. These differences are precisely of the same nature as differences of colour. A piece of coloured glass permits rays of certain colours to pass and absorbs others. The difference between red and violet rays is the same in kind as the difference between dark and luminous rays.

CONVECTION.

242. In liquids and gases the transference of heat from one portion to another is chiefly effected by *convection*, that is to say, by the intermixture of hotter and colder portions. Liquids and gases are very bad conductors, and heat applied to their upper portions descends with extreme slowness, so that water, for example, may be brought to the boil in the upper part of a test tube while the lower part contains unmelted ice. But if heat is applied to their lower portions, these ascend as they become heated, in consequence of the diminution of their specific gravity by expansion, and colder portions descend to be heated in turn. The currents thus produced can be rendered visible by putting saw-dust into the liquid. If a spirit-lamp is held under a beaker of water into which oak saw-dust has been dropped, the saw-dust will be observed to ascend immediately over the

lamp, and, after reaching the top, to descend again at a distant part. It will then flow inwards towards the place where the heat is applied, and will again ascend, thus manifesting the existence of a continual circulatory movement in the liquid.

243. The ascent of smoke and hot air up a chimney is another example of a convection current. In order that a chimney may draw well, the opening at the bottom must be of suitable size. If it is too large, a quantity of cold air from the room will accompany the hot air which has passed through the fire, and will, by its greater specific gravity, retard the ascent. A tall chimney draws better than a short one, because the force to which the ascent is due is the difference between the weight of the chimneyful of hot air and the weight of an equal volume of cold air.

244. Winds are (at least in their origin) to be regarded as convection currents. If any part of the earth's surface becomes hotter than surrounding parts, the tendency is for air to flow in to it below, to ascend over it, and then to spread outwards above. It is thus that the sea-breeze is produced, every afternoon, at places near the coast in intertropical countries, the ground having been heated by the sun's rays to a higher temperature than the sea; while about twelve hours later, the reverse effect, a land breeze, occurs, the ground having fallen at night to a lower temperature than the sea.

The monsoons of the East Indies may be described as land and sea breezes, occurring on a large scale, and governed not by the diurnal but by the annual movement of the sun.

245. The trade-winds are due, in the first instance, to the excessive heating of the equatorial parts of the earth —both land and sea. An ascent of air is thus produced over the equatorial belt, and colder air flows in from both sides. Owing to the earth's rotation the winds thus produced are not due north and due south, but north-east and south-east. The air which has ascended in the equatorial

regions spreads out on both sides, and finally descends (perhaps in the polar regions) to be again drawn towards the equator. The whole atmosphere is thus in a continual state of circulation.

246. A similar circulation is doubtless taking place in the ocean, but much more slowly, because water is much less expansible by heat than air. The currents which are familiar to navigators in many parts of the ocean are merely superficial, and are probably due to wind acting on the surface of the water. The general circulation to which we now refer is on a much larger scale, but is probably too slow to attract attention unless special means are used for detecting it. Numerous observations made in recent years show that the bottom of the ocean, even in equatorial regions, is at a temperature not much higher than that at which fresh water freezes. This cold water has doubtless found its way along the depths of the sea from the polar regions, while a general flow from equator to poles is taking place nearer to the surface. In connection with oceanic circulation it is to be noted that sea water (unlike fresh water), when cooled, continues to contract until it reaches its freezing point.

HYGROMETRY.

247. If water is put into a vessel containing perfectly dry air, and the vessel is then closed so that no air or vapour can enter or escape, a portion of the water will evaporate and mix with the air. The quantity of vapour that will be formed depends upon the temperature of the inclosure, and very little or not at all upon the density of the air within it, being nearly the same for vacuum as for air at atmospheric density. The denser the air the more slowly will the vapour be formed and the longer will be the time required to attain the condition of equilibrium in which evaporation ceases, but the quantity of vapour

requisite for equilibrium is practically unaffected by the presence of the air. This important law, which holds for other liquids as well as for water, was discovered by Dalton, who also showed that the pressure of the mixture of air and vapour thus produced is equal to the pressure which the air would exert if the vapour were removed, *plus* the pressure which the vapour would exert if the air were removed, or, as the law is usually expressed, the *pressure of the mixture is the sum of the pressures due to the air and vapours separately.* This second law is identical with that which determines the pressure of a mixture of two gases.

248. For any given temperature there are a certain definite density and a certain definite pressure which the vapour tends to attain if in contact with its liquid, and which it cannot exceed. These are called the ***maximum density*** and ***maximum pressure for that temperature***, and the word ***maximum*** is frequently omitted when no misunderstanding is likely to occur.

249. The maximum pressure increases very rapidly as the temperature increases. Its value at 10° C. is double of its value at 0°, and its value at 120° is double of its value at 100°.

250. The temperature corresponding to any given maximum pressure is called the boiling point for this pressure. The boiling point accordingly rises as the pressure increases.

When a kettle half full of cold water is placed on the fire, the space above the water contains a mixture of air and vapour until the water comes to the boil, the pressure of the mixture being all the time equal to or slightly greater than that of the external air. When the boiling point is reached, the pressure of the vapour alone is equal to that of the external air. As the boiling proceeds the boiling point is slightly exceeded, and the steam, having a slightly greater pressure than the external air, is able to expel it and take exclusive possession.

251. Let a little water be boiled in a Florence flask,

and while it is boiling let the flask be corked air-tight, and the source of heat removed. We thus obtain an inclosure containing only water and vapour of water. Ebullition will immediately cease, but if the flask be now plunged into cold water (so as to be completely immersed) ebullition will recommence and will continue for some time. This experiment, which was contrived by Franklin, is explained by the diminution of temperature and pressure in the vapour by immersion in cold water. The water in the flask having much greater capacity for heat than the vapour, is less rapidly cooled, and its temperature is accordingly higher than the boiling point corresponding to the diminished pressure. The same effect may be obtained by inverting the flask and squeezing a wet sponge over it, as in Fig. 65.

Fig. 65.—Franklin's Experiment.

252. In like manner water which is a little below the boiling point in the external air, if placed in an open vessel under the receiver of an air-pump, will be made to boil by a few strokes of the pump.

253. As the boiling point of water in the open air is an index of the pressure of the air, an observation of the boiling point is equivalent to an observation of the barometer. A thermometer with very long degrees finely subdivided and reading only from about 95° to 100°, is often employed, in conjunction with a small apparatus i-

which water can be boiled, for determining the heights of mountains. It is equivalent to a barometer, and has the advantage of great portability. The bulb of the thermometer is not immersed in the water but in the steam, the vessel being so contrived that the steam must circulate round the thermometer before escaping through a small opening into the air. It is found that the temperature of the steam given off by boiling water is steadier than that of the water. Water, especially if contained in very clean glass vessels, often rises above the theoretical boiling point, then bursts violently into ebullition, and falls again.

254. The variations of density, pressure, and temperature in aqueous vapour—as long as none of it passes into the liquid form—are connected by the same laws as those of a gas; that is to say, its pressure is proportional to its density when the temperature is constant, and to the absolute temperature when its density is constant. The density of aqueous vapour at any temperature is ·622 (or nearly $\frac{5}{8}$) of the density of air at the same temperature and pressure. For example, the weight of a cubic foot of dry air at 10° C., under the pressure of 30 inches of mercury, being 547 grains, and the maximum pressure of vapour at 10° C. being ·361 of an inch of mercury, the maximum weight of a cubic foot of vapour at 10° C. is

$$\cdot 622 \times \frac{\cdot 361}{30} \times 547 = 4\cdot 1 \text{ grains.}$$

255. The air in which we live always contains aqueous vapour. When the density of this vapour approaches the maximum for the existing temperature, we feel a sense of oppression, and a slight exertion suffices to throw us into a sensible perspiration. On the other hand, when the ratio of the actual to the maximum density is unusually small, evaporation takes place with excessive rapidity, rendering the skin dry and liable to crack or peel off.

256. The ratio of the actual to the maximum density

of vapour, or, what is the same thing, the ratio of the actual to the maximum pressure of vapour is technically called the *humidity* or the *relative humidity* of the air. It is usually expressed as a percentage. For example, "humidity 75 per cent." means that the quantity of vapour present in a given space is $\frac{3}{4}$ of the greatest quantity that could exist in that space at the actual temperature.

257. The *dew-point* is a name used to denote the lowest temperature to which the air can be cooled down (at constant pressure) without depositing some of its vapour in the shape of liquid water. If a body colder than the dew-point be in contact with the air, dew will be deposited upon it.

258. The actual pressure of vapour in the air is equal to the maximum pressure at the dew-point; so that, if the dew-point be determined by direct observation, we can find the pressure of vapour in the air by a table of maximum pressures. This observation is usually made by employing a thin vessel of polished silver or of black glass, which is gradually cooled by means of the evaporation of ether from its interior, the vapour of the ether being conveyed away through a tube, so as not to mingle with the air around the vessel. A thermometer stands with its bulb in the vessel, and is read off at the moment when dew is first observed. The evaporation of the ether is then checked, the temperature rises, and the thermometer is read again at the moment when dew disappears. The mean of the two temperatures thus observed is assumed to be the dew-point. The temperature of the air should, at the same time, be observed by another thermometer.

259. When the dew-point is below the freezing-point, the deposit upon surfaces colder than the dew-point is not liquid water, but ice in the shape of hoar-frost.

Water or ice loses substance (by evaporation) if its temperature be above the dew-point, and gains substance (by the deposition of dew or hoar-frost) if its temperature be below the dew-point. Hence the dew-point may be defined as the temperature which water or ice must have

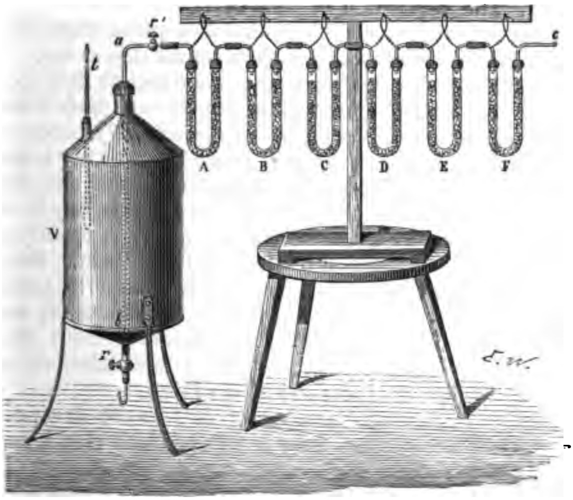
that it may neither lose nor gain substance when exposed to the air.

260. Water or ice at the temperature of the dew-point tends to rise in temperature, for it is receiving heat from the air (which is at a higher temperature) and is not losing heat by evaporation. That temperature which water or ice can retain unchanged when freely exposed to a current of air is called the *temperature of evaporation*, and is higher than the dew-point, but not so high as the temperature of the air. It is almost as sharply defined as the dew-point, but is difficult to observe in calm weather. It is commonly observed by means of a thermometer whose bulb is tied up in muslin fed with water by a cotton wick. Another thermometer, at the distance of a few inches, is employed to give the temperature of the air at the same time. The two thermometers are commonly mounted on a single stand, and the entire instrument is called the *wet-and-dry-bulb hygrometer*. When the air is very dry (especially if it be also very warm) the difference between the readings of the two thermometers is great. When the vapour in the air is at maximum density the two readings are the same, provided that the temperature of the air is steady.

261. The mass of vapour in a given volume of air can be directly measured by the chemical hygrometer (Fig. 66). V is a vessel called an *aspirator*, which at the beginning of the experiment is full of water. A B C D E F are *drying tubes*, that is, glass tubes containing pieces of pumice soaked in sulphuric acid, which has a strong affinity for water and rapidly abstracts aqueous vapour from moist air. A large portion of the water in the aspirator is allowed to run out through the tap *r*, and is measured. To fill its place, air enters at *c* and flows through the series of drying tubes, depositing nearly all its moisture in the first tube F, and the remainder in the two next tubes E, D. The other tubes A, B, C are merely intended to prevent moisture reaching the first tubes from the aspirator. At the conclusion of the experiment the tubes

F, E, D are weighed. They were also weighed at the beginning of the experiment, and the increase observed is the weight of the moisture which has been deposited. The volume of external air which contained this moisture

Fig. 66.



is very nearly equal to the volume of water which has run out of the aspirator. The thermometer t gives the temperature of the aspirator, which must be observed when great accuracy is required. It is usually assumed that the air in the aspirator, at the conclusion of the experiment, is charged with vapour at the maximum density for this temperature.

262. Whichever kind of hygrometer be employed, it is necessary and sufficient to observe two elements in order to obtain a complete determination of the state of the air as regards moisture. For example, if the temperature of the air and the dew-point are observed, we can calculate

the density and pressure of vapour, the relative humidity, and the temperature of evaporation. The two elements most commonly observed are the temperature of the air and the temperature of evaporation. The dew-point and other elements above-mentioned are then inferred, by the aid of a book of tables constructed for the purpose.

263. The deposition of dew or hoar-frost upon grass and other bodies in the open air takes place when these bodies attain a lower temperature than the dew-point. On clear nights, bodies at the earth's surface radiate heat to the cold regions which lie beyond the atmosphere, and if there is little or no wind they thus fall considerably below the temperature of the air a few feet above them. Dew is most copiously deposited on the best radiators and the worst conductors, because these are bodies which attain the lowest temperature by nocturnal radiation. A cloudy sky checks radiation, and thus hinders the loss of heat. Strong winds keep up the temperature of the ground by mixing up the different strata of air and bringing continually fresh portions into contact with the ground. Both these agencies accordingly keep up the temperature of the ground, and hinder the deposition of dew.

264. When a large mass of air falls to the dew-point, any further loss of heat from it will produce cloud, mist, rain, or snow.

Rain is measured by an instrument called a rain-gauge, consisting usually of a funnel-shaped collector leading to a suitable holder below. The mouth of the funnel must be horizontal, and the depth of rain in any shower is assumed to be equal to the volume of water collected in the gauge divided by the area of the mouth. The average annual rainfall in Great Britain, taking one place with another, is about 30 inches.

EXAMPLES IN HEAT.

XXI.—Scales of Temperature. Arts. 177, 178.

1. The difference between the temperatures of two bodies is 30° F. Express this difference in degrees Cent. and in degrees Réau.

2. The difference between the temperatures of two bodies is 12° C. Express this difference in degrees Réau. and in degrees Fahr.

3. The difference between the temperatures of two bodies is 25° R. Express this difference in the Cent. and Fahr. scales.

4. Express the temperature 70° F. in the Cent. and Réau. scales.

5. Express the temperature 60° C. in the Réau. and Fahr. scales.

6. Express the temperature 30° R. in the Cent. and Fahr. scales.

7. Air expands by ·00366 of its volume at the freezing-point of water for each degree Cent. By how much does it expand for each degree Fahr.?

8. The temperature of the earth increases by about one degree Fahr. for every 50 feet of descent. How many feet of descent will give an increase of 1° Cent.?

9. The mean annual range of temperature at a certain place is 100° F. What is this in degrees Cent.?

10. Lead melts at 326° C., and in melting absorbs as much heat as would raise 5·37 times its mass of water 1° C. What numbers will take the place of 326 and 5·37 when the Fahrenheit scale is employed?

11. Show that the temperature $-40°$ C. and the temperature $-40°$ F. are identical.

12. What temperature is expressed by the same number in the Fahr. and Réau. scales?

XXII.—Expansion of Solids and Liquids.

Arts. 179–191.

The following coefficients of expansion can be used:—

Linear.		Cubical.	
Steel,	·0000116	Glass,	·000024
Copper,	·0000172	Mercury,	·000179
Brass,	·0000188	Alcohol,	·001050
Glass,	·0000080	Ether,	·001520
		Sulphuret of carbon,	·001140

The Centigrade scale of temperature is employed.

1. The correct length of a steel chain for land measuring is 66 ft. Express, as a decimal of an inch, the difference between the actual lengths of such a chain at 0° and 20°.

2. One brass yard-measure is correct at 0° and another at 20°. Find, as a decimal of an inch, the difference of their lengths at the same temperature.

3. A lump of copper has a volume 258 cc. at 0°. Find its volume at 100°.

4. A glass vessel has a capacity of 1000 cc. at 0°. What is its capacity at 10°?

5. A weight-thermometer contains 462 gm. of a certain liquid at 0° and only 454 gm. at 20°. Find the mean relative expansion per degree between these limits.

6. A weight-thermometer contains 325 gm. of a liquid at zero and 5 gm. run out when the temperature is raised to 12°. Find the mean coefficient of apparent expansion.

7. If the coefficient of relative expansion of mercury in glass be $\frac{1}{6500}$, what mass of mercury will overflow from a weight-thermometer which contains 650 gm. of mercury at 0° when the temperature is raised to 100°?

8. The capacity of the bulb of a thermometer together

with as much of the stem as is below zero is ·235 cc. at 0°, and the section of the tube is $\frac{1}{2000}$ sq. cm. Compute the length of a degree (1), if the fluid be mercury; (2), if it be ether.

9. The bulb, together with as much of the stem as is below the zero-point, contains 3·28 gm. of mercury at zero, and the length of a degree is 1 cm. Compute the section of the tube, the density of mercury being about 13·6.

10. Show that when ice begins to form on a deep pond the great body of the water will be at 4°.

XXIII.—Volume, Pressure, and Temperature of Gases. Arts 192–195.

1. What will be the volume at 300° of a quantity of gas which occupies 1000 cc. at 0°, the pressure being the same?

2. What will be the volume at 400° of a quantity of gas which occupies 1000 cc. at 100°, the pressure being the same?

3. What will be the pressure at 30° of a quantity of gas which at 0° has a pressure of 30 in. of mercury, the gas being confined in a close vessel whose expansion may be neglected?

4. A thousand cc. of gas at 1·0136 of an atmosphere are allowed to expand till the pressure becomes one atmosphere, and the temperature is at the same time raised from its initial value 0° to 100°. Find the final volume.

5. A gas initially at volume 4500 cc., temperature 100°, and a pressure represented by 75 cm. of mercury, has its pressure increased by 1 cm. of mercury and its temperature raised to 200°. Find its final volume.

6. At what temperature will the volume of a gas at a pressure of one atmosphere be 1000 cc., if its volume at temperature 0° and pressure 1·02 of an atmosphere be 1200 cc.?

XXIV.—Thermal Capacity. Arts. 196–208.

The following values of specific heat can be used:—

Iron,	·1098	Mercury,	·033
Copper,	·0949	Alcohol,	·548
Platinum,	·0335	Ether,	·529
Sand,	·215	Air, at constant pressure,	·2375
Ice,	·504		

1. 17 parts by mass of water at 5° are mixed with 23 parts at 12°. Find the resulting temperature.

2. 200 gm. of iron at 300° are immersed in 1000 gm. of water at 0°. Find the resulting temperature.

3. 60 gm. of platinum at 500° are immersed in 400 gm. of mercury at 10°. Find the resulting temperature.

4. Find the specific heat of a substance 80 gm. of which at 100°, when immersed in 200 gm. of water at 10° give a resulting temperature of 20°.

5. 16 parts by mass of sand at 75°, and 20 of iron at 45° are thrown into 50 of water at 4°. Find the temperature of the mixture.

6. 300 gm. of copper at 100° are immersed in 700 gm. of alcohol at 0°. Find the resulting temperature.

7. If the length, breadth, and height of a room are respectively 6, 5, and 3 metres, how many gramme-degrees of heat will be required to raise the temperature of the air which fills the room by 20°, the pressure of the air being constant, and its average density ·00128 gm. per cubic centimetre?

8. Find the thermal capacities of mercury, alcohol, and ether per unit volume, their densities being respectively 13·6, ·791, and ·716.

XXV.—Latent Heat. Arts. 209–214.

The following values of latent heat can be used:—

In Melting.		In Evaporation at Atmospheric Pressure.	
Water,	80	Steam,	536
Lead,	5·4		

1. Find the result of mixing 5 gm. of dry snow at 0° with 23 gm. of water at 20°.

2. Find the result of mixing 6 lbs. of dry snow at 0° with 7 of water at 50°.

3. Find the temperature obtained by introducing 10 gm. of steam at 100° into 1000 gm. of water at 0°.

XXVI.—Thermodynamics. Arts 215–221.

1. From what height must mercury fall in order to raise its own temperature 1° Fahr. by the destruction of the velocity acquired, supposing no other body to receive any of the heat thus generated, and from what height must it fall to raise its temperature 1° Cent.?

2. What is the greatest proportion of the heat received by an engine at 200° that can be converted into mechanical effect, if the heat which is given out from the engine is given out at the temperature 10°?

3. If a perfect engine gives out heat at 0°, at what temperature must it take in heat that half the heat received may be converted?

XXVII.—Conduction. Arts. 228–233.

(*Units the centimetre, gramme, and second.*)

1. How many gramme-degrees of heat will be conducted in an hour through each sq. cm. of an iron plate ·02 cm. thick, its two sides being kept at the respective temperatures 225° and 275°, and the mean conductivity of the iron between these temperatures being ·12?

2. Through what thickness of copper would the same amount of heat flow as through the ·02 cm. of iron in the preceding question, with the same temperatures of its two faces, the mean conductivity of the copper between these temperatures being unity?

3. How much heat will be conducted in an hour through each sq. cm. of a plate of ice 2 cm. thick, one side of the ice being at 0° and the other at −3°; and what

volume of water at 0° would be converted into ice at 0° by the loss of this quantity of heat?

4. How much heat will escape in an hour from the walls of a building, if their area be 80 sq. metres, their thickness 20 cm., their material sandstone of conductivity ·01, and the difference of temperature between outside and inside 15°? What quantity of carbon burned per hour would generate heat equal to this loss?

ANSWERS.

XXI.—Ex. 1. 16°$\frac{2}{3}$ C., 13°$\frac{1}{3}$ R. Ex. 2. 9°$\frac{2}{9}$ C., 21°$\frac{3}{5}$ F. Ex. 3. 31°$\frac{1}{4}$ C., 56°$\frac{1}{4}$ F. Ex. 4. 21°$\frac{1}{9}$ C., 16°$\frac{8}{9}$ R. Ex. 5. 48° R., 140° F. Ex. 6. 37°$\frac{1}{2}$ C., 99°$\frac{1}{2}$ F. Ex. 7. ·00203. Ex. 8. 90 ft. Ex. 9. 55°$\frac{5}{9}$ C. Ex. 10. 619°, 9·666. Ex. 12. $-25·6$.

XXII.—Ex. 1. ·184 in. Ex. 2. ·0135 in. Ex. 3. 259·33 cc. Ex. 4. 1000·24 cc. Ex. 5. ·000881. Ex. 6. ·001302. Ex. 7. $\frac{650}{66} = 9·85$ gm. Ex. 8. (1) ·073 cm., (2) ·703 cm. Ex. 9. ·000432 sq. cm.

XXIII.—Ex. 1. 2098 cc. Ex. 2. 1804 cc. Ex. 3. 33·3 in Ex. 4. 1385 cc. Ex. 5. 5631 cc. Ex. 6. $-50°$.

XXIV.—Ex. 1. 9°·02. Ex. 2. 6°·44. Ex. 3. 74°·8. Ex. 4. $\frac{5}{16} = ·3125$. Ex. 5. 10°. Ex. 6. 6°·91. Ex. 7. 547200. Ex. 8. ·449, ·433, ·379.

XXV.—Ex. 1. Water at 2°$\frac{1}{7}$. Ex. 2. 1$\frac{5}{8}$ part snow, 11$\frac{3}{8}$ water, all at zero. Ex. 3. Water at 6°·3.

XXVI.—Ex. 1. 25·5 ft., 45·9 ft. Ex. 2. $\frac{100}{273} = ·4$ nearly. Ex. 3. 273°.

XXVII.—Ex. 1. 1080000. Ex. 2. $\frac{1}{6}$ cm. $\doteq ·166\dot{6}$ cm. Ex. 3. 11·88 gm.-deg., ·149 cc. Ex. 4. 21600000 gm.-deg., 2700 gm.

LIGHT.

265. Rays of light, as long as they proceed in a uniform medium, are straight.

When rays travelling through air fall upon the surface of a sheet of glass they are broken up into three parts. One part enters the glass, another part is reflected in such a manner as to produce images similar to those seen in a looking-glass, and a third portion, which is extremely small if the glass is well polished, is scattered in all directions. It is by this last portion that the surface of the glass is rendered visible.

When the incident rays are nearly normal most of the light enters the glass. When they are nearly parallel to the surface of the glass most of the light is reflected.

When light falls upon a bright surface of silver, quicksilver, or speculum metal, most of it is reflected, whatever be the direction of incidence.

We shall first discuss the laws of reflection.

REFLECTION.

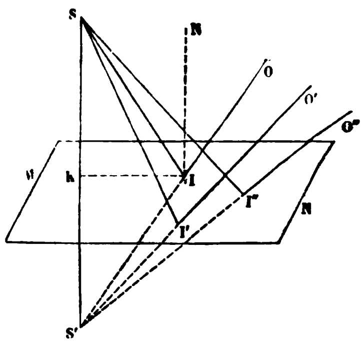
266. The point at which a ray falls upon a surface is called the *point of incidence*, and the ray is said to be *incident on* the surface at this point.

The angle which the incident ray makes with the normal at the point of incidence is called the *angle of incidence*, and the angle which the reflected ray makes with the normal is called the *angle of reflection*. The fundamental law of reflection is that *the normal at the point of incidence bisects the angle between the incident and the reflected ray*. In other words, *the angle of reflection is equal to the angle of incidence and in the same plane with it.*

267. This law leads at once to the explanation of the formation of images by a plane mirror.

Let S, Fig. 67, be a small object in front of the mirror. Draw S K normal to the mirror, and produce, making K S′ = S K. Then all rays from S to the mirror will be reflected in lines which, if produced backwards, pass through S′. For if I be the point of incidence of one of these rays it is easily shown by comparing the right-angled triangles S K I, S′ K I that the angles K S I, K S′ I are equal. But if I N be the normal at I, and S′ I be produced to O, the angle of incidence S I N is equal to the alternate angle K S I, and the angle N I O is equal to S S′ I. Therefore the normal bisects the angle between S I and I O, and I O must be the direction of the reflected ray. The appearance presented to an eye receiving the reflected rays is consequently the same as if they came from a real object at S′.

Fig. 67.



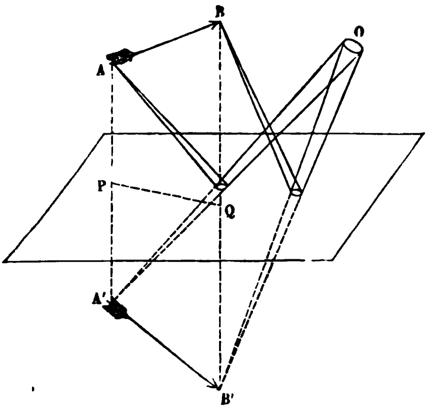
The point S′ is called the image of the point S with respect to the mirror, and the two points considered as a pair are called conjugate foci with respect to the mirror. Conjugate foci with respect to a plane mirror lie at equal distances on opposite sides of the reflecting surface, and on the same normal.

Rays diverging from S are reflected from the mirror as if they came from S′. Rays converging to S′ from the region in front of the mirror are reflected to meet in S.

268. The image of an object of finite size is the aggregate of the images of its several points; thus A′ B′, Fig. 68, is the image of A B. The figure shows the construc-

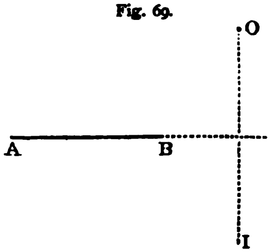
tion for finding the two points A′, B′, and also contains a representation of the pencils (that is, cones or conical bundles) of rays which enter the pupil of an observer's eye at O, and convey to it the impression oi images at A′ and B′.

Fig. 68.



In order that a mirror may form an image of an object, it is not necessary that the object should be directly in front of the mirror. Thus if O (Fig. 69) be an object in front of the plane of the mirror A B, but not in front of the mirror itself, its image I is found by dropping a perpendicular from O on the plane of the mirror, and then producing to a point at the same distance on the other side of the plane.

Fig. 69.



In order that an observer may be able to see the image of an object in a mirror, some part of the mirror must be between his eye and the image. Thus if A B (Figs. 70, 71) be a plane mirror, O an object, and I its image, the image will be visible to an eye situated anywhere within the space P A B Q.

269. Images of Images.—When rays which have been reflected from one mirror fall upon a second, a

second image is formed which is the image of the first image.

Thus, if we have two plane mirrors A A, B B (Fig. 72) facing each other, and an object O situated between their

Fig. 70.

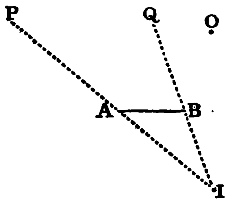
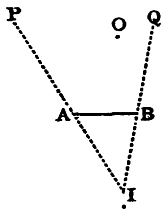
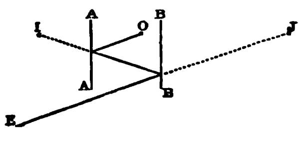


Fig. 71.



planes, an image of O will be formed by the mirror A A at I, and an image of this image will be formed by the mirror B B at J, as far behind the plane of B B as I is in front of it. This second image will be visible to an eye at E, the rays which form it having taken the course O A B E. In drawing a figure to represent this course, we must first find the positions of the two images, then rule a line from the observer's eye to the second image,

Fig. 72.



then from the point where this line cuts the second mirror rule a line to the first image, and finally from the point where this line cuts the first mirror rule a line to the object.

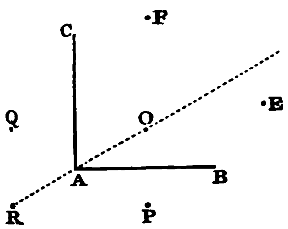
In the circumstances here supposed, a great multitude of images will in reality be formed. An image of the second image will be formed by the mirror A A, an image of this third image by B B, and so on indefinitely. Moreover, a direct image of O will be formed by B B, an image of this image will be formed by A A, and so on. Upon

the whole there will be two images formed by first reflections, two by second reflections, two by third reflections, and so on without end if the mirrors be accurately parallel, all the images being ranged in one straight line normal to the mirrors and passing through O. As, however, the reflected light is always less in quantity than the incident light, the first images will be less bright than the object, the second images less bright than the first, and each order of images less bright than its predecessor. There is thus a practical limit to the number of images which can be seen.

270. Shape of Images formed by Plane Mirrors.—An image formed by a single reflection from a plane mirror bears the same relation of shape to the object which a left hand bears to a right hand. This reversal of shape is itself reversed when a second reflection occurs, so that the second image is similar to the object itself.

271. Mirrors at Right Angles.—Let A B, A C (Fig. 73) be two plane mirrors facing inwards at right angles to each other, and O an object within the angle between them. The mirror A B gives an image of O at P. The mirror A C gives an image of O at Q. Also, the mirror AB gives an image of Q at R, and the mirror A C gives an image of P at the same point R.

Fig. 73.



The images P and Q are seen by means of light which has been once reflected. The image R is seen by means of light which has been twice reflected. If the observer's eye is situated at E within the angle O A B, the rays by which R is seen are reflected first from A C and then from A B. If the eye is at F within the angle O A C, the opposite order holds.

272. Mirrors inclined at 60°.—The series of successive images formed by two plane mirrors are always

situated on the circumference of a circle whose centre lies at the intersection of the planes of the mirrors. If the angle between the mirrors is an aliquot part of 360°, the last image (which is formed by one more reflection than any of the others) consists of two coincident images, as has already been shown to be the case when the angle is a right angle.

If, for example, the angle A C B (Fig. 74) between the mirrors be 60°, there will be two images formed by first reflections, and these will lie in the sectors (each of 60°) B C A′, A C B′, which are adjacent to the sector contained by the mirrors; there will be two images formed by second reflections, and these will lie in the sectors adjacent to those which contain the first images, namely, in A′ C B″, B′ C A″. Finally, by third reflections two coincident images will be formed in the remaining sector A″ C B″; and as this image is behind the planes of both mirrors it yields no fourth reflection.

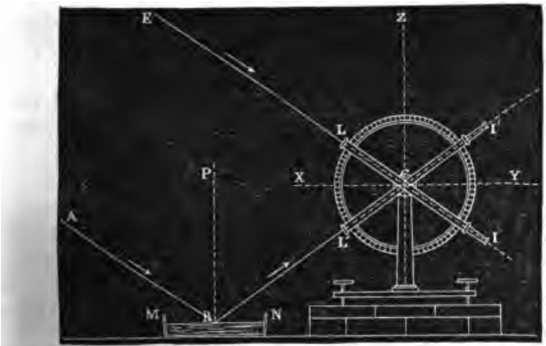
Fig. 74.

Two mirrors attached together at an angle which is a submultiple of 360° are sometimes used by designers for obtaining symmetrical patterns, and a very beautiful optical toy called the kaleidoscope was constructed on the same principle by Sir David Brewster.

273. Artificial Horizon.—The angular distance between a star and its image as seen by reflection from a horizontal surface of mercury is double of the altitude of the star. For if A B (Fig. 75) be a ray from the star incident on the mercury, and B L′ the reflected ray, these

rays make equal angles with the vertical line B P, and are therefore equally inclined to a horizontal plane. By

Fig. 75.

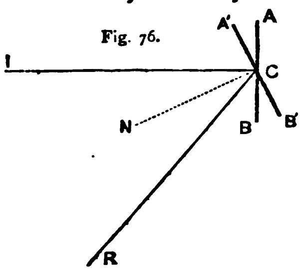


reason of the enormous distance of the star, the incident ray A B may be regarded as parallel to a ray from the star to the observer's eye.

This method of observation is much used in astronomy, and the dish of mercury which is employed is called an *artificial horizon.*

274. Deviation Produced by Rotating a Mirror.—A ray normally incident on a mirror is reflected back upon itself. If we now rotate the mirror through any acute angle θ round an axis in its own plane, the normal will make an angle θ with the same incident ray. Hence the reflected ray will make an angle 2θ with the incident ray instead of coinciding with it. When a mirror is rotated round any axis perpendicular to a given incident ray, the reflected ray is

Fig. 76.



turned through double the angle of rotation. In the annexed figure (Fig. 76) I C is the given incident ray, A B the position of the mirror when this ray is normal, A′ B′ the position of the mirror when turned through an angle θ; C N is the normal to the mirror in this position and C R the reflected ray. Each of the angles I C N, N C R, is equal to θ.

275. When a ray is reflected first from one and then from the other of two parallel mirrors, its initial and final courses are parallel. If, now, the second mirror be rotated in the plane of reflection, the final direction of the ray will be changed by double the angle of rotation. This result may also be stated in the following form:—When a ray is twice reflected, in such a manner that its whole course lies in one plane, the angle between its initial and final directions is double of the angle between the mirrors.

This is the principle of the sextant, an instrument of immense use in navigation.

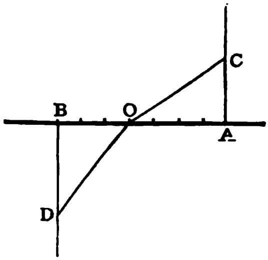
REFRACTION.

276. When a ray in air falls normally upon the surface of a piece of glass or a sheet of water, it continues its course in the same straight line in the second medium. When, on the other hand, it is incident obliquely, it bends at an angle, its course in the water or glass being more nearly normal than its course in air. A ray coming from water or glass into air likewise bends at an angle, the bending in this case being away from the normal instead of towards it. In both cases the angle made with the normal is smaller in the denser than in the rarer medium, and the same bent line represents the course of the ray taken from either end. The bending of a ray which occurs when it passes out of one medium into another is called *refraction*, and the acute angles which the incident and the refracted ray make with the normal at the point of incidence are called the *angles of incidence and refraction* respectively.

These two angles lie in the same plane, and their difference —being the measure of the amount of bending—is called the *deviation*. The angles of incidence and refraction increase together, and the deviation increases with them.

277. The following construction gives the course of a ray in passing from air into water.

Fig. 77.



Let B A (Fig. 77) be the section of the surface of the water made by the plane of incidence, and let C O be the incident ray. Lay off O A equal to 4 and O B to 3 equal parts, and draw the perpendiculars A C, B D, the former meeting the incident ray in any point C. In the perpendicular B D (regarded as of indefinite length) find a point D such that O D = O C. Then O D will be the refracted ray.—The ratio of O A to O B in this construction is called the *index of refraction*. Its value is almost exactly $\frac{4}{3}$ for rays passing out of air into water, and is about $\frac{3}{2}$ for rays passing out of air into the less dense kinds of glass. For rays passing out of water into such glass it will be the quotient of $\frac{3}{2}$ by $\frac{4}{3}$, that is $\frac{9}{8}$, and it is always possible to calculate in this way the index out of one medium into a second, when the index out of some third medium into each of the two is given. (See arts. 284, 285.)

When the order of the two media is reversed the index is also reversed; thus, the index from water into air is $\frac{3}{4}$, from glass (of the kind above mentioned) into air $\frac{2}{3}$, and from this glass into water $\frac{8}{9}$.

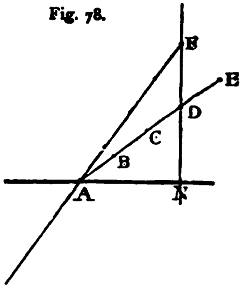
278. The following is another construction leading to precisely the same result as that above given:—

If the ray in air is given, lay off upon it 4 equal parts A B, B C, C D, D E (Fig. 78), starting from the point of incidence A. Through D draw a normal N D F, and on it find a point F whose distance from A is equal to A E.

Then F A produced will be the path of the ray in the water.

If the ray in water is given (Fig. 78), lay off upon its production in air 4 equal parts, and drop a normal F N from the extremity of the 4th part. Upon F N find a point D such that A D shall be equal to 3 of these parts. A D will be the path of the ray in air.

Fig. 78.



LIMITING ANGLE OF REFRACTION.

279. Limit of the Foregoing Constructions.—The construction given in the preceding article becomes impossible when A N is greater than A D, in other words when A N is greater than $\frac{3}{4}$ of A F.

If we draw a normal N F and make A F equal to $\frac{4}{3}$ of A N, we obtain a limiting direction F A, such that rays in water which are more nearly normal than F A will be refracted into air according to the above construction, whereas those which are more nearly parallel to the surface will follow a different law. These latter, in fact, are not refracted into air at all, but are reflected back into water just as if the surface of the water were a mirror.

We are led to the same limiting angle by the construction of art. 265, for that construction ceases to be possible when the point C coincides with A, in which case O D being equal to O A is $\frac{4}{3}$ of O B.

The angle which the ray in water in the limiting position makes with the normal is called the *critical angle*. As the ray in air moves from the normal to the tangential direction, the ray in water moves from the normal to the critical direction.

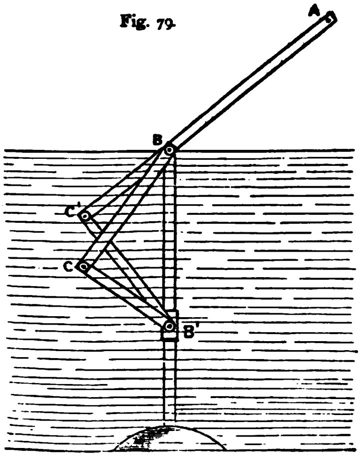
280. The ratio $\frac{AN}{AD}$ in art. 266 is called the *sine* of the angle A D N, and the ratio $\frac{OB}{OD}$ in art. 265 is called the

sine of the angle O D B. Hence the sine of the critical angle is equal to the index of refraction from the denser to the rarer medium, which in the case of water and air is $\frac{3}{4}$.

The general law of refraction may be stated in the following words: *the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant* when the two media are given, and is called *the index of refraction from the first medium to the second.* This law is usually referred to as the *law of sines.*

281. The corresponding variations of position of the incident and refracted rays are best exhibited to the eye by Sir George Airy's apparatus (Fig. 79), which can be used either as a moving diagram or as a means of experimentally verifying the law.

Fig. 79.



B′ is a slider travelling up and down a vertical stem. A C′ and B C are two rods pivoted on a fixed point B of the vertical stem. C′ B′ and C B′ are two other rods jointed to the former at C′ and C, and pivoted at their lower ends on the centre of the slider. B C is equal to B′ C′, and B C′ to B′ C. Hence the two triangles B C B′, B′ C′ B, are equal to one another in all positions of the slider, their common side B B′ being variable, while the other two sides of each remain unchanged in length though altered in position. The ratio $\frac{BC}{CB'}$ or $\frac{B'C'}{C'B}$ is made equal to the index of refraction

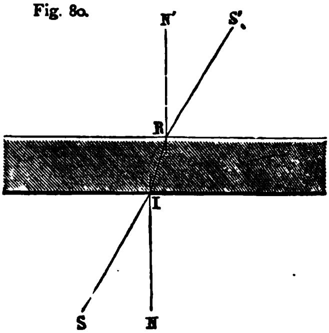
of the liquid in which the observation is to be made. For water, this ratio will be $\frac{4}{3}$. Then if the apparatus is surrounded with water up to the level of B, A B C will be the path of a ray, and a stud at C will appear in the same line with studs at A and B.

282. The reflection of rays which are beyond the critical angle can be observed by putting a spoon into a glass of water and holding it above the level of the eye. That part of the spoon which is below the surface will be seen reflected in the surface as in a highly polished mirror. Indeed, the reflection of rays which are beyond the critical angle is the most complete kind of reflection known, exceeding that of the best metallic mirrors. It is almost absolutely total, and is often distinguished by the name of *total reflection*, or *total internal reflection*.

SUCCESSIVE REFRACTIONS.

283. Refraction through a Plate.—A piece of glass or other transparent material, with two parallel faces, is called a plate. A ray is bent nearer to the normal in entering a plate from air, but is bent away from the normal again in emerging into air at the opposite face, and its course after leaving the plate is parallel to its course before entering it. For if S I R S′ (Fig. 80) be the path of the ray, the angle of refraction at I is equal to the angle of incidence at R, and therefore the angle of incidence at I is equal to the angle of refraction at R.

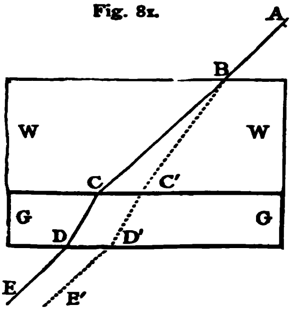
Fig. 80.



284. When two or more plates are superimposed, the

course of a ray in traversing the system is such that the part of the ray within each plate is parallel to what it would have been if the ray had been allowed to enter this plate direct from air. Thus, if G G (Fig. 81) be a plate of glass, a ray A C incident upon its upper face in air will take such a course as A C D E, D E being parallel to A C. If the glass be covered with a plate of water W W, the path of the ray will be changed to A B C′ D′ E′, C′ D′ being parallel to C D, and therefore D′ E′ to D E or A B. These conclusions can be deduced from the rule above given (art. 265) for calculating the index of refraction from water into glass; or conversely from the observed fact that the course of a ray before incidence on a system of superimposed plates is parallel to its course after emergence, the rule for calculation can be established.

Fig. 81.



285. Thus if we have two superimposed plates, and if the index of refraction from air into the first is μ_1, and from air into the second is μ_2, let α be the angle which a ray before incidence and after emergence makes with the common normal, and α_1, α_2, the angles which the parts of it in the first and second plates respectively make with the normal. Then denoting the sines of these angles by the usual notation, sin α, sin α_1, sin α_2, we have by the law of refraction, at the first surface,

$$\text{Sin } \alpha = \mu_1 \sin \alpha_1.$$

In like manner we have by the law of refraction at the last surface,

$$\text{Sin } \alpha = \mu_2 \sin \alpha_2.$$

Hence we deduce

$$\mu_1 \sin \alpha_1 = \mu_2 \sin \alpha_2, \text{ or } \frac{\sin \alpha_1}{\sin \alpha_2} = \frac{\mu_2}{\mu_1};$$

that is, the index of refraction out of the first plate into the second is $\frac{\mu_2}{\mu_1}$.

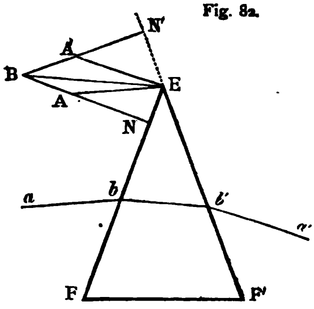
286. Refraction through a Prism.—A *prism* in optics means a transparent body having two plane faces not parallel to one another. It would more aptly be called a wedge. A ray entering at one of these faces and emerging at the other is changed in direction, being deflected towards the thick end of the wedge. A plane perpendicular to the *edge* in which the two faces meet, or would meet if produced, is called a *principal plane*, and discussions regarding refraction through prisms are usually confined to rays lying in principal planes. This limitation is to be understood in all that follows.

The angle formed by the meeting of the two faces, or by the meeting of their planes produced, is called the *refracting angle* of the prism, or simply the *angle* of the prism.

287. The following construction, which is based on art. 266, gives the course of a ray refracted through a prism. We shall assume the index of refraction to be $\frac{3}{2}$, but the construction is easily adapted to any other index.

Let F E F′ (Fig. 82) be the refracting angle. Draw E B parallel to the ray in the prism, and make its length equal to 3 parts of any convenient scale. Drop normals B N, B N′, on the two faces or their planes produced, and find points A, A′ upon them such that E A and E A′ are equal to 2 parts. Then A E and A′ E are parallel to the two rays in air, and the whole course of the ray,—as $abb'a'$,—can be traced by making its three portions parallel to A E, B E, and A′ E respectively.

Fig. 82.



288. We have here started by assuming a particular direction for the ray in the prism. If we assume a particular direction for the incident ray, we must first draw E A, then the normal N A produced indefinitely beyond A, then find B, and complete the construction as above.

289. Whatever the index of refraction may be from the external medium into the prism we are to make the ratio of B E to A E or A′ E equal to this index, which we will denote by μ.

290. Minimum Deviation.—The angle A E A′ is the deviation produced by the prism, and can be shown to be least when the ray in the prism makes equal angles with the two faces, in which case the whole course of the ray is symmetrical. This is usually the actual course which a ray is made to take through a prism in optical researches, and the prism is in this case said to be in the "position of minimum deviation."

291. Prism of Small Angle.—When the angle of the prism is small (say not more than 20°) the minimum deviation, or the deviation in any position not exceedingly unsymmetrical, is approximately $\mu - 1$ times the angle of the prism.[1]

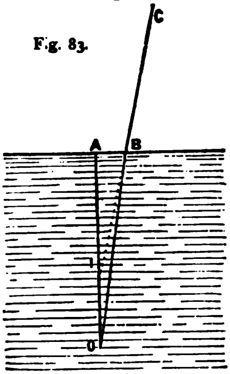
FORMATION OF IMAGES.

292. Image of a Point by Refraction at a Plane Surface.—A small object under water, as seen by an observer in air, appears in a position different from that which it really occupies; and its apparent position varies with the position of the observer. We shall confine our calculations to the case in which the observer's eye is directly opposite to the object,—that is to say, the case in which the eye and the object are on the same normal to the refracting surface. The image seen in this case is called the *geometrical image.*

[1] Since B E is μ.A E, angle E A N is approximately μ.E B N, therefore A E B is $(\mu - 1)$ E B N. Similarly A′ E B is $(\mu - 1)$ E B N′. Therefore, by addition, A E A′ is $(\mu - 1)$ N B N′. But A E A′ is the deviation, and N B N′ is the angle of the prism.

Let O (Fig. 83) be the position of the object, O A the normal, O B C the path of a nearly normal ray, and let B C, the part of this ray in air, be produced to meet the normal in I. Then, since the index of refraction out of water into air is $\frac{3}{4}$, we have (art. 278) $\frac{BI}{BO} = \frac{3}{4}$. But, as B moves up to A, I B and I A become equal, as also do O B and O A. Hence if we take upon A O a point I such that

Fig. 83.



$$AI = \tfrac{3}{4} AO,$$

I will be the position of the geometrical image. It is the point in which the normal is cut by such rays as B C produced, when the angle A O B is very small. The pupil of the eye is not a mere point; it receives a small pencil of rays, and all the rays by which the point O is seen would, if produced backwards, cut each other almost precisely in the point I thus found, the pupil being supposed to be so situated that all these rays are nearly normal to the surface of the water.

As the eye moves away from the normal the image approaches nearer to the surface, but so long as the eye is near the normal this movement of the image is exceedingly slow. If the eye is moved with uniform velocity from one side of the normal to the other, the image moves with a velocity which continually diminishes till the normal is reached, becomes zero at the normal, and then again gradually increases. This is a general property of geometrical images, whether formed by refraction or reflection, a geometrical image being defined as one which is formed by normal and nearly normal rays.[1]

[1] The case of images formed by reflection at plane surfaces (art. 267), the image in this case being unaffected by movement of the eye, is altogether special and exceptional.

293. The points O and I are called conjugate foci. A small pencil of nearly normal rays diverging from O emerges as if it came from I, and a small pencil of nearly normal rays converging in air towards I will be refracted to meet in O.

The preceding investigation can be adapted to rays refracted from any medium into another through a plane surface. A small pencil of nearly normal rays from a point in the first medium will emerge into the second as if they came from a point I in the first, and this point I lies on the normal through O, at a distance μ.A O from the surface, that is, we have

$$A I = \mu . A O,$$

A being the point where the normal meets the surface, and μ the index of refraction from the first medium into the second. Thus, if the object be in air, and the other medium be water, the distance of the image from the surface will be $\frac{4}{3}$ of the distance of the object.

REFLECTION FROM SPHERICAL SURFACES.

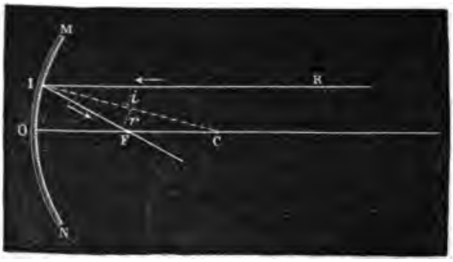
294. By a *spherical mirror* is meant a mirror whose reflecting surface is a portion of the surface of a sphere. The radius of the sphere is called the *radius of curvature* of the mirror, and the centre of the sphere is called the *centre of curvature.* If the outline of the mirror is circular, that point of the reflecting surface which is equidistant from all points of the circumference is called the *centre of the mirror.*

295. Let R I (Fig. 84) be a ray incident upon the concave spherical mirror M N. Draw the radius C I. Then the angles of incidence and reflection, marked i and r in the figure, are equal; and if C O be a radius of the mirror drawn parallel to the incident ray, and meeting the reflected ray in F, the angle O C I is equal to the angle of incidence. Therefore, in the triangle F I C we have the angles F I C and F C I equal, and the side F I

equal to the side F C. But, when the arc O I is small compared with the radius, I F is sensibly equal to O F; in this case, therefore, O F is sensibly equal to F C, that is, the reflected ray very nearly bisects the radius parallel to the incident ray.

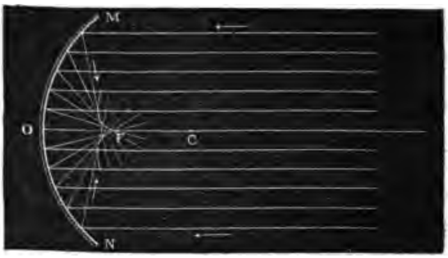
If O is the centre of the mirror, the radius O C indefi-

Fig. 84.



nitely produced is called the *principal axis* of the mirror, and the limiting position of the point F, that is to say, the point midway between O and C, is called the *principal focus*. In general the principal focus of any mirror or

Fig. 85.



lens is the focus for rays parallel to the axis. The distance of the principal focus from the mirror or lens is called the *focal length*.

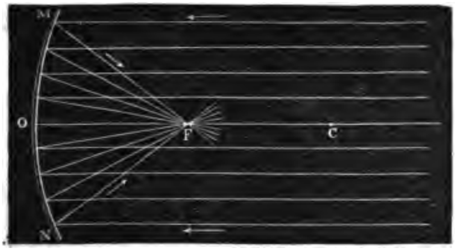
295. When the arcs O M, O N are not small in com-

parison with the radius, the rays reflected from the outer portions of the mirror will fall sensibly short of the middle point of O C, which is denoted by F in Fig. 85. If the point of incidence be supposed to travel with uniform velocity along the arc M O from M to N, the intersection of the reflected ray with O C will move towards F with velocity gradually diminishing to zero.

All the reflected rays in the plane of the figure touch a certain curve, shown in the figure, called a *caustic*. The focus F is the point at which the two branches of this curve meet, and is therefore said to be at the *cusp* of the caustic.

The angle subtended at the centre of curvature by the whole arc M O N is called the *angular aperture* of the

Fig 86.

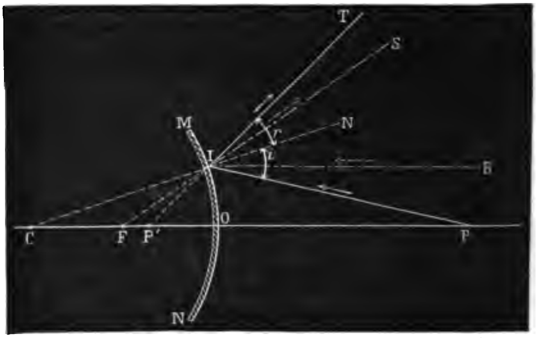


mirror. In elementary optics it is usual to suppose the angular aperture of a spherical mirror to be small, and to regard all the reflected rays as accurately meeting in the principal focus (Fig. 86) when the incident rays are parallel to the principal axis.

297. Convex Mirror.—In like manner it can be shown that a ray R I (Fig. 87) incident upon a convex spherical mirror M N parallel to one of its radii O C will, if the arc O I is small compared with the radius, be reflected in a direction I S, which, if produced backwards, passes through a point F sensibly coincident with the

middle point of O C. If O is the centre of the mirror, the middle point of O C is still called the *principal focus*,

Fig. 87.



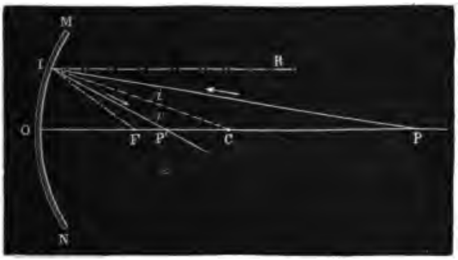
and is said to be a *virtual* focus, because it is not the reflected rays themselves but only their imaginary productions that meet in it.

298. Image of the Sun formed by Concave Mirror.—When the axis of a concave spherical mirror is directed towards the sun, an image of the sun is formed midway between the mirror and the centre of curvature. Any given point of the sun's disc sends parallel rays to all parts of the mirror, and these are reflected to a point which is the image of that particular point of the sun's disc; the aggregate of all these images makes up the image of the sun. They all lie on a spherical surface described about the centre of curvature, with a radius half that of the mirror, and the line joining any point of the sun to its own image passes through the centre of curvature. From this last property it follows that the diameters of the sun and of its image subtend equal angles at the centre of curvature. The magnitude of this angle is about half a degree.

As all the rays of the sun reflected by the mirror are

collected into the area occupied by the image, the heating effect at the focus is very intense when the mirror is large

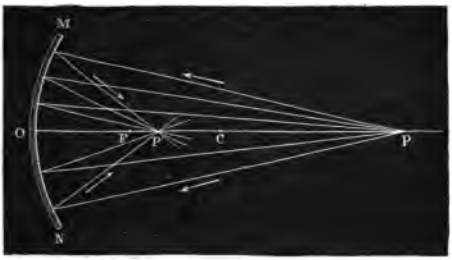
Fig. 88.



compared with the image. Hence the name *focus*, literally meaning *hearth* or fireplace.

299. Conjugate Foci of Concave Mirror.— Let rays fall upon a concave mirror (Fig. 88) from a point P at a distance from the mirror greater than the

Fig. 89.



radius of curvature. Draw the line P C O through the centre of curvature C to meet the mirror in O. Then any incident ray P I will be reflected so as to make the angle of reflection *r* equal to the angle of incidence *i*, and it can be shown that if the angular aperture be small, all

11

the reflected rays will meet sensibly in one point P′ (Fig. 89). It can also be shown that

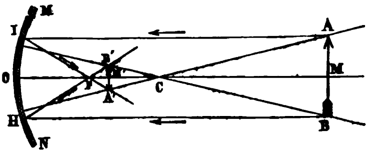
$$\frac{1}{OP} + \frac{1}{OP'} = \frac{1}{OF},$$

F being the middle point of O C.

Conversely, rays incident upon the mirror from P′ will sensibly meet after reflection in the point P. Hence P and P′ are called conjugate foci, each of these points being the image of the other. As P moves away to an unlimited distance, P′ moves to F; and as P moves to C, P′ moves to meet it in C.

300. Construction for Conjugate Foci.—Let A B (Fig. 90) be an object at a distance from the mirror greater than the radius of curvature, and let C be the centre of curvature. Then we know that the incident ray A C H will be reflected back upon itself, because it is incident normally. Hence the image of the point A must lie somewhere on the line A C H.

Fig. 90.



If A I be any other ray from A to the mirror, and C O be a radius drawn parallel to A I, we know that the reflected ray must pass through F, the middle point of C O. The point A′ in which this reflected ray meets the line A C H must be the image of A.

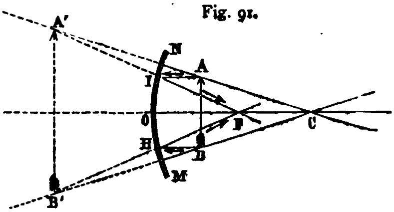
A verification will be obtained by drawing a line from A through F to meet the mirror. We know that all rays from F to the mirror will be reflected parallel to F C. It will be found that the reflected ray so drawn passes through the point A′ found as above.

If the object A B be an arc of a circle described about C as centre, its image will be an arc of a circle also described about C, and the two arcs will subtend equal angles at C.

If the object A B be a straight line short compared with the radius of curvature and perpendicular to the radius O C produced, its image A′ B′ will be sensibly straight and will also be perpendicular to O C. Hence, when the point A′ (the image of A) has been found as above, B′, the image of B, can be found by drawing B C and producing it to meet a line drawn through A′ perpendicular to O C.

301. Construction for Virtual Image.—If the object A B be between the principal focus and the mirror M N (Fig. 91) the incident ray A I parallel to any radius C O will be reflected to F, the middle point of C O, also the ray A N incident in the direction of the radius C A will be reflected back upon itself; hence A′, the image of A, will be at the intersection of the lines C A and F I produced. It will be behind the mirror, and the image will be virtual.

Fig. 91.



Conversely, rays converging to A′ from the region in front of the mirror will, after reflection from the mirror, meet in A.

302. Comparison of the Two Cases.—Comparing Figs. 90 and 91 we have the following results:—

Case 1. One of two conjugate foci is between the principal focus and the centre of curvature, and the other is beyond the centre of curvature.

Case 2. One is between the principal focus and the mirror, and the other is behind the mirror.

In the first case, since the lines A A′, B B′ cross one another between the object and the image, the image is inverted. In the second case the lines A A′, B B′ do not cross each other between the object and the image, and the image is erect.

In both cases, since the triangle C A B is similar to the triangle C A′ B′, the linear dimensions of object and image are directly as their distances from C, the centre of curvature.

It can be shown that the distances of conjugate foci from the centre of curvature are directly as their distances from the mirror itself.

When the object is near to C or to O the image is approximately equal to it in size. When the object is near to F the image is very large and very distant.

303. Real and Virtual Images.—In the first case the image is *real*, that is to say, it is formed by rays which actually arrive at it. In the second (the object being in front of the mirror and the image behind the mirror), the rays by which the image is seen have not actually come from the place where the image appears to be, but the rays by which any point of the image is seen would meet in that point if produced backwards. This is the characteristic of a *virtual image*. The images seen in an ordinary looking-glass are virtual. The names *real* and *virtual* are also applied to foci; thus in case 1 of last article both foci are real, in case 2 one is real and the other virtual.

304. Real Image thrown on Screen.—Let P and P′ (Fig. 92) be conjugate foci, both real, with respect to the concave mirror M N. Then if a candle-flame be placed at P and a small screen held at P′, as represented in the figure, an inverted image of the flame will be visible on the screen. Conversely if the candle-flame be at P′ its inverted image will be thrown upon a screen held at P. An image capable of being thrown on a screen is necessarily a real image, but real images are not always sufficiently bright to be rendered visible in this way. Without a screen, an observer will see a real image in mid-air, if he places himself in such a position that the image is between his eye and some part of the mirror, and is at a suitable distance for distinct vision.

305. Reflecting Telescopes. Herschelian.—In all reflecting telescopes the rays from the object fall in the first instance upon a large concave mirror, technically

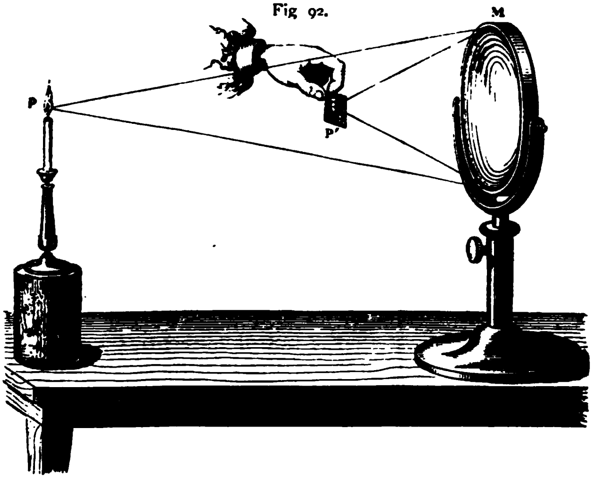


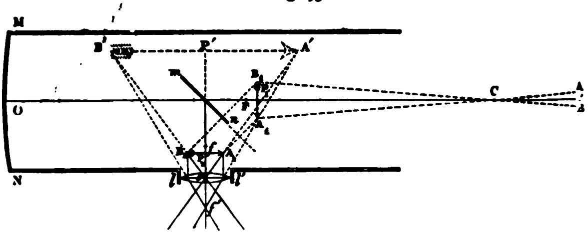
Fig 92.

called a *speculum*. In the Herschelian telescope there is no second mirror; the observer simply views the image through a lens or combination of lenses called an eye-piece, and it is necessary that the eye, the image, and some part of the mirror should be in one straight line. If the axis of the mirror were directed towards the object, it would be necessary for the observer's eye to be also on the axis, and his head would obstruct the rays on their way to the mirror. To avoid this difficulty the mirror must be set somewhat obliquely, a circumstance which is very unfavourable to good definition, and this form of telescope is therefore very little used.

306. Newtonian Telescope.—In the Newtonian telescope (Fig. 93) a small plane mirror *mn* is interposed at an angle of 45° to the common axis of the tube and the

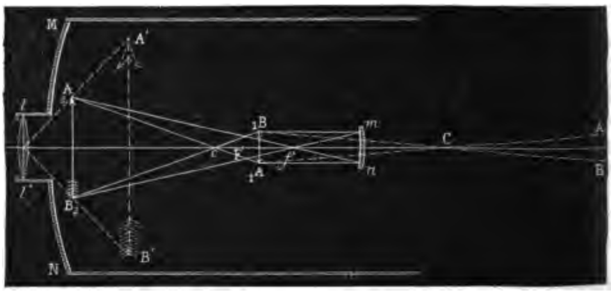
plane speculum M N in such a position as to intercept the reflected rays on their way to form the first image $A_1 B_1$,

Fig. 93.



and reflect them to one side, where they accordingly form a second image $A_2 B_2$ of the same size as the first. To construct for its position we drop perpendiculars from A_1 and B_1 on the plane of the mirror and produce them to an equal distance in front of the mirror, as explained in art. 267. The plane of the image $A_2 B_2$ is parallel to the

Fig. 94.



axis of the telescope, and the eye-piece ll' is inserted in the side of the telescope, directly opposite to it. This construction is almost universal in large reflecting telescopes intended for astronomical purposes.

307. Gregorian Telescope.—The favourite reflecting telescope for observing terrestrial objects is the Gregorian (Fig. 94). The large speculum M N is perforated in its centre with a hole large enough to look through, and behind this hole is the eye-piece *l l'*. The rays reflected from the large speculum are intercepted by a second much smaller concave mirror *m n* after forming the first image $_1A\,_1B$, which lies between the centre of curvature and principal focus of the second mirror. Hence an enlarged image $A_2\,B_2$ is formed at a greater distance from *m n*; and to this image the eye-piece is directed.

The first image $_1A\,_1B$ is an inverted image of the object. The second image $A_2\,B_2$ is an inverted image of the first,

Fig. 95.

and is therefore an erect image of the object. The observer, looking through the eye-piece, accordingly sees an erect image in this telescope; and there is the further convenience, for terrestrial purposes, that he looks in the direction of the object. Fig. 95 is a sectional view

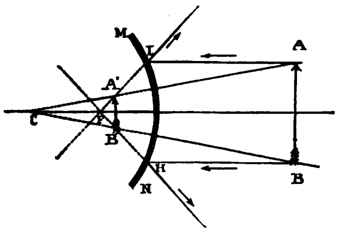
of the complete instrument. V is the focussing screw, which moves the small mirror *m*.

308. Specula of Telescopes.—The mirrors, or *specula*, employed in telescopes are not of glass, coated at the back with amalgam, like ordinary looking-glasses, but are either composed of an alloy of copper and tin called speculum metal, or of glass coated in front with silver.

In large telescopes they are usually not spherical but approximately parabolic—that is, of the form generated by the rotation of a parabola about its axis. If a mirror is truly parabolic, all rays incident upon it parallel to the axis are accurately reflected to the focus if it be concave, or from the focus if it be convex.

309. Conjugate Foci for Convex Mirrors.—Let A B (Fig. 96) be an object in front of a convex spherical mirror M N. The ray A C incident in the direction of the centre of curvature is reflected back upon itself, and the ray A I, incident in any other direction, is reflected as if it came from F, the middle point of the parallel radius; hence the point A′ in which F I and C A intersect is the image of A. The image A′ B′ of an object A B anywhere in front of a convex mirror is erect, virtual, and smaller than the object, and lies behind the mirror at a distance less than half the radius of curvature. When the object is close in front of the mirror the image is close behind the mirror, and as the object moves away to infinite distance the image moves back to the middle point of the radius. The image, being always between the centre of curvature and the object, is always smaller than the object.

Fig. 96.



310. Application to Cassegranian Telescope.

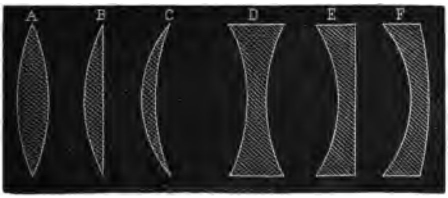
—As rays incident on the mirror from A are reflected as if they came from A′, so also rays converging in front of the mirror to A′ are reflected to A. This principle is employed in the Cassegranian telescope, which like the Gregorian has a perforated speculum with the eye-piece behind it. The rays reflected from the large speculum are intercepted on their way to form a real image A′ B′ by a small convex mirror M N, and after this second reflection they form a real and larger image A B just in front of the eye-piece. The second image is erect as compared with the first, and therefore inverted as compared with the object.

LENSES.

311. Lenses are of two principal kinds, convex and concave. A convex lens is very similar in its properties to a concave mirror, and a concavė lens to a convex mirror.

The three lenses A, B, C (Fig. 97) are all classed as

Fig. 97.



convex. The first is called double convex, the second plano-convex, and the third a convex meniscus (μηνισκος, a crescent).

The three lenses D, E, F are all classed as concave. The first is called double-concave, the second plano-concave, and the third a concave meniscus.

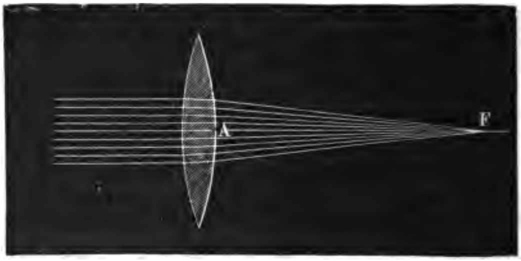
The two forms of meniscus can be distinguished by

comparing the thickness in the middle with the thickness at the outside. If the middle is the thickest part, the lens is to be regarded as convex; if it be the thinnest part, the lens is concave.

Both faces of a lens are usually spherical or approximately spherical; and in elementary optics the greatest thickness of a lens is supposed to be small in comparison with the radii of curvature of the faces.

312. Principal Focus.—Rays incident upon a con-

Fig. 98.



vex lens parallel to its axis (Fig. 98) are refracted through it in such a manner that after emerging they converge approximately to one point F, called the *principal focus;* and the principal focus is said to be real.

Rays incident upon a concave lens parallel to its axis emerge from it as if they came from one point (F, Fig. 99), called the principal focus; and the principal focus in this case is said to be virtual.

The distance of the principal focus from the lens (in either case) is called the *focal length* of the lens.

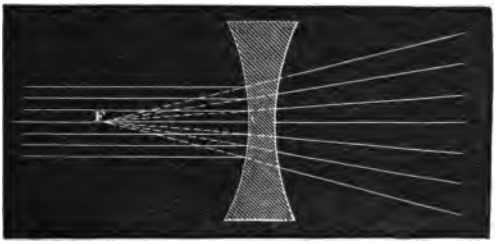
313. The formula for finding the focal length f when the index of refraction μ and the radii of curvature r_1, r_2, of the two faces of the lens are given, is

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right),$$

which is applicable to all the six forms of lens above

described, when proper attention is paid to the signs of the quantities involved.

Fig. 99.



f is to be considered positive when the principal focus is real, negative when it is virtual.

r_1 is to be reckoned positive if the first face is convex, negative if the first face is concave.

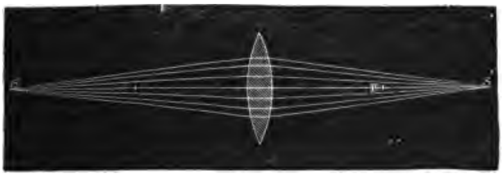
r_2 is to be reckoned positive if the second face is convex, negative if concave.

μ denotes the index of refraction from the external medium into the lens.

As rays may be incident upon either face of a lens, there are two principal foci, at equal distances from the lens, on opposite sides of it.

314. Conjugate Focal Distances.—Rays inci-

Fig. 100.



dent upon a convex lens from a principal focus emerge parallel to the axis, and rays incident upon a concave

lens in lines converging to a principal focus emerge parallel to the axis.

Rays incident upon a convex lens from a point (S, Fig. 100) on or near its axis at a greater distance than its focal length, will approximately converge to a real focus (S′) on the other side, and if p and p' denote the distances of S and S′ respectively from the lens, they will satisfy the equation

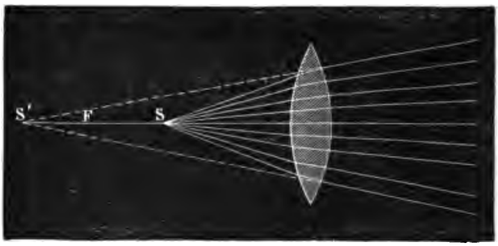
$$\frac{1}{p}+\frac{1}{p'}=\frac{1}{f},$$

f denoting the focal length of the lens.

Conversely, rays incident upon the lens from S′ will converge approximately to S. The two points S and S′ are called *conjugate foci.*

315. Rays incident upon a convex lens from a point S (Fig. 101), nearer than the principal focus, diverge after

Fig. 101.



emergence as if they came from a more distant point S′ on the same side of the lens as S. The points S and S′ in this case are still called conjugate foci, S being a real and S′ a virtual focus, and their distances from the lens will satisfy the formula of the preceding article if the distance of the virtual focus from the lens be counted negative.

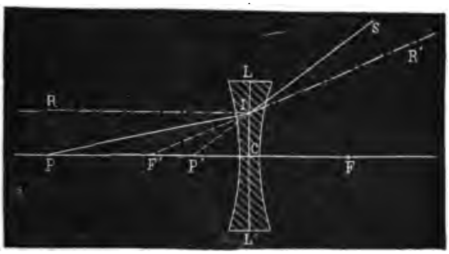
Rays converging on the other side of the lens towards S′ will, after emergence, converge to S.

316. Applications.—When we employ a pocket lens to magnify a small object, we hold the lens at a distance from it less than the focal length, and thus form a virtual image on the same side as the object, but at a greater distance. If the eye is close to the lens, which is the best position when high magnification is desired, the distance of the virtual image should be equal to the distance at which the observer would hold a small object which he wished to examine carefully with the naked eye.

When a lens is used as in a magic lantern or a camera obscura, for throwing the image of a real object on a screen, the image must be real, and the circumstances are those discussed in art. 302.

317. Conjugate Foci of Concave Lens.—If rays fall upon a concave lens (Fig. 102) from a point P

Fig. 102.



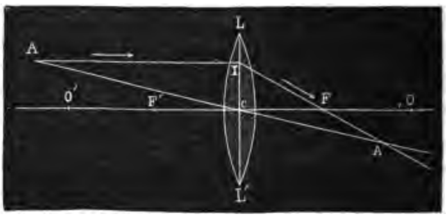
at any distance whatever, they will emerge as a pencil diverging from a nearer point P′ on the same side of the lens. The distance C P′ is not only less than C P, but is also less than the focal length C F′ or C F. If P is very distant (and lies on the axis) P′ will nearly coincide with the principal focus on the same side. If P is very near to the lens, P′ will be still nearer. Its movements are governed by the equation

$$\frac{1}{CP'} - \frac{1}{CP} = \frac{1}{CF}.$$

318. Optical Centre of a Lens.—Every lens has a point whose properties are similar to those of the centre of curvature of a mirror. It is called the *centre* or *optical centre* of the lens. It lies on the axis, and if the lens be equi-convex or equi-concave it is the middle point of the thickness. If the lens be plano-convex or plano-concave it lies upon the convex or concave face, and if the lens be a meniscus it lies outside the lens. It possesses the property that every straight line drawn through it makes equal angles with both faces of the lens; whence it follows that any ray which in its passage through the lens traverses this point (or would traverse it if produced in the case of a meniscus) will, like a ray traversing a plate, have the same direction after emergence as before entrance,—that is to say, the emergent ray will be parallel to the incident ray. In elementary optics, the thickness of a lens is supposed to be negligible in comparison with the other distances which are discussed, and the emergent ray, which is really parallel to (and at a very small distance from) the incident ray, is regarded as coincident with it; in other words, *the course of every ray which traverses the centre of a lens is regarded as straight.*

319. Rules for Magnitude and Inversion.—By tracing rays from two points of an object, through the centre of a lens, to the corresponding points of the image,

Fig. 103.

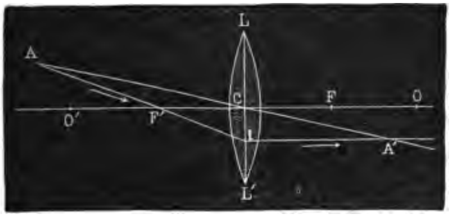


it can be shown, as in the case of mirrors, that the linear dimensions of object and image are directly as their dis-

tances from the centre. Also that the image will be inverted when it lies on the opposite side of the centre from the object, and will be erect when it lies on the same side.

320. Construction for Image.—When the centre and focal length of a lens are given, the position and size

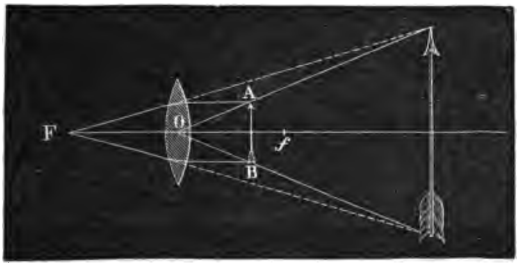
Fig. 104.



of the image of a given object can be found by a construction analogous to that which we have already employed in connection with mirrors.

Let A (Fig. 103) be a point not lying on the axis. The ray A I incident parallel to the axis will pass through the

Fig. 105.

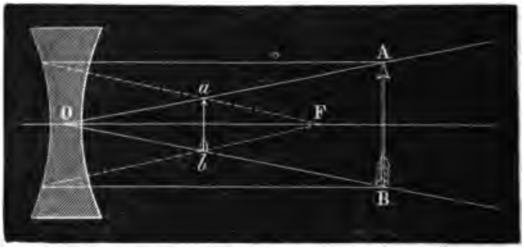


principal focus F, and the ray A C through the centre of the lens will continue its course without deviation. Hence

the point A′ in which the two lines I F, A C intersect when produced, is the image of A.

Or again (Fig. 104), the ray A F′ I which passes through the principal focus F′ will emerge parallel to the axis.

Fig. 106.



Hence A′ the image of A will be the intersection of the line A C produced with I A′ drawn parallel to the axis.

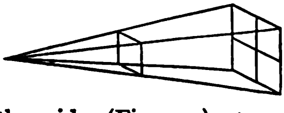
In these figures both the conjugate foci are real.

The construction when one is virtual is exhibited in Figs. 105, 106, in both of which A B is the object, O the centre of the lens, and F one of the two principal foci.

MAGNIFICATION.

321. Apparent Size.—When one object, as seen by an eye in a given position, exactly covers another, they are said to have the same apparent size; their images upon the retina of the eye are, in fact, equal.

Fig. 107.



When we speak of the apparent size of one object as being double that of another, it is necessary to specify whether we are speaking of length or area. A square 1 inch in the side (Fig. 107) at a given distance from the eye has the same apparent size as a square 2 inches in the

side at double the distance. The 2-inch square can be divided into four squares each of 1 inch. Each of these has half the apparent linear magnitude of the nearer square, and has one-fourth of the apparent area. An object is said to be magnified *n* diameters when its apparent linear dimensions are increased *n* fold, and this is the usual mode of specifying magnifying power.

322. Magnification by a Lens or Simple Microscope.—In the attempt to increase the apparent size of small objects by bringing them near the eye, we are practically limited by the nearest distance of distinct vision. Near-sighted persons have an advantage over others in viewing small objects by the naked eye, inasmuch as they are able to bring them nearer without loss of distinctness.

When we use a convex lens in the ordinary way to magnify a small object, a virtual image is formed (as in Fig. 105) at a greater distance from the lens, and therefore from the eye, than the object itself. The object and its image subtend the same angle at the centre of the lens, and, if the eye is close to the lens, subtend practically the same angle at the eye. If, then, the image is at the smallest distance of distinct vision, the object will be so near that it could not be seen distinctly without the lens. When a lens is thus used, its magnifying effect simply depends upon the power which it thus gives of bringing an object nearer without losing distinctness of vision. When we view an object through a convex lens not close to the eye, the image subtends, at the eye, a larger angle than the object, as is evident from Fig. 105. This result depends on the circumstance that the image is more distant than the object. When a concave lens is employed, the image (*a*, *b*, Fig. 106) is nearer than the object, and therefore, if the eye is not close to the lens, subtends a smaller angle than the object.

323. Spectacles.—The spectacles usually worn by aged persons are intended to remedy the inability to focus the eye for near objects. The remedy consists in

forming images of near objects at greater distances than the objects themselves.

Spectacles for short-sighted persons are concave, and form images of objects at smaller distances than the objects themselves.

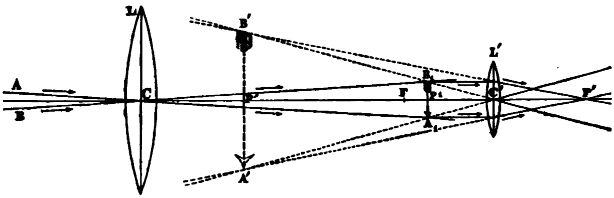
If x denote the distance at which a person who needs spectacles finds by trial that he can see most distinctly, and D the distance at which a book would be held by a person who has no defect of vision, the focal length f of the spectacles which should be used is given by the formula—

$$\frac{1}{f} = \frac{1}{x} - \frac{1}{D} \text{ for short-sight.}$$

$$\frac{1}{f} = \frac{1}{D} - \frac{1}{x} \text{ for long-sight.}$$

324. Astronomical Refracting Telescope.— The telescope specially designated the *astronomical* telescope, or the astronomical refractor, consists essentially

Fig. 108.



of two convex lenses L and L′ (Fig. 108), called respectively the *object-glass* and *eye-piece*, or the *objective* and the *ocular*, the former being much larger than the latter and having much greater focal length.

The object-glass forms an inverted image A_1 B_1 at or a little beyond its principal focus, and the eye-piece produces a virtual image A′ B′ of this image. C and C′ are the centres of the two lenses, F and F′ are the two principal foci of the eye-piece, and the dotted lines indicate the construction (as in Fig. 105) for the virtual image.

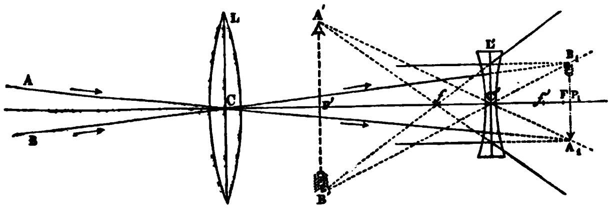
325. Its Magnifying Power.—The magnifying power of a telescope is estimated by comparing the angle under which a distant object is seen in the telescope with the angle under which it is seen by the naked eye; that is, by comparing the angle A′ C′ B′ or $A_1 C' B_1$, in the fig. of last article, with A C B or its equal $A_1 C B_1$.

But the angles $A_1 C' B_1$ and $A_1 C B_1$, being the angles subtended by the real image at the centres of the two lenses, are approximately in the inverse ratio of the distances of this image from the two centres, and these distances again are approximately equal to the focal lengths. Hence the magnifying power as regards apparent linear dimensions is approximately equal to

$$\frac{\text{focal length of objective}}{\text{focal length of ocular}}.$$

326. Galilean Telescope.—In the telescope invented by Galileo, the ocular L′ (Fig. 109) is a concave

Fig. 109.



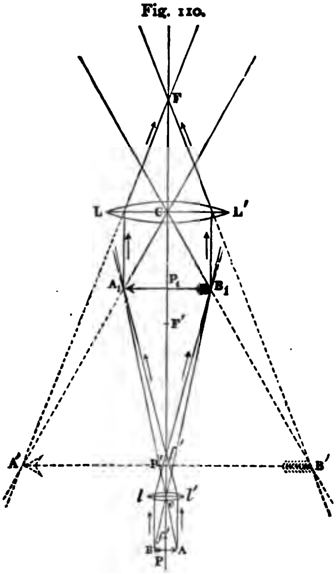
lens, and intercepts the rays before they have arrived at the focus of the objective. It is in advance of this focus by a distance rather greater than its own focal length, and it forms a virtual image A′ B′ of the real image $A_1 B_1$ which would be formed if the ocular were removed out of the way, this real image being at or a little beyond the principal focus of the objective L. Let f and f' be the principal foci of the ocular. Then a ray on its way towards A_1 in a direction parallel to the common axis emerges from the ocular as if it came from f; and the point A′ in

which this emergent ray produced backwards intersects the production of $A_1 C'$, is the image of A_1.

The magnifying power is approximately the ratio of the focal lengths of the two lenses, just as in the case of the "astronomical" telescope. The same rule also holds for the Herschelian and Newtonian telescopes described in arts. 294, 295. Objects are seen erect in the Galilean telescope, whereas in the "astronomical" telescope they are seen inverted. The Galilean construction is employed for opera-glasses. In the ordinary terrestrial telescope the image is rendered erect by the introduction of some additional lenses besides the two which belong to the "astronomical" telescope.

327. Compound Microscope.—The compound microscope in its simplest form consists essentially of two convex lenses, one of which, called the objective, is of short focal length, and is placed at a distance slightly exceeding this length from the object to be viewed. A greatly-enlarged real image is formed on the other side at a distance which is very large compared with the focal length, and this image is magnified by means of the other lens, called the ocular or eye-piece. In Fig. 110, A B is the object, $l\,l'$ the objective, $f' f'$ its two principal foci, $A_1 B_1$ the real image, here drawn by the construction of art. 320, Fig. 103; L L′ is the ocular, F and F′

Fig. 110.



its principal foci, and A′ B′ the virtual image which is seen by the observer, this image being here constructed according to art. 320, Fig. 105.

Compound microscopes are usually furnished with several objectives of different focal lengths—the most usual being from an inch down to $\frac{1}{8}$ of an inch; and large telescopes are usually furnished with several oculars. In both cases the shortest focal length gives the highest magnifying power.

328. Brightness of Image.—The brightness of the image seen in a telescope or microscope can never exceed the brightness of the object as seen by the naked eye, except in the case of bodies which, like stars, appear mere luminous points. In the majority of cases, especially when high magnification is obtained, there is great loss of brightness as compared with that presented to the naked eye.

When a telescope or microscope is directed towards the sky or any bright back-ground, a real image of the objective is formed by the ocular, and may be received on a piece of thin paper held at a little distance—perhaps $\frac{1}{8}$ or $\frac{1}{4}$ of an inch—behind the ocular. If this bright spot is smaller than the pupil of the observer's eye, the difference represents so much loss of brightness. If it is larger than the pupil, there is no gain of light, because the light that falls outside the pupil does not contribute to vision. In telescopes, the diameter of the objective divided by the diameter of this spot is equal to the magnifying power. A large objective, therefore, permits a comparatively high magnifying power to be employed without rendering the bright spot smaller than the pupil. When the bright spot is as large as the pupil, the brightness would be the same as to the naked eye, were it not that lenses transmit only a part of the light which falls upon them—another part being reflected from their surfaces,—and that specula only reflect a portion of the incident light.

329. Cross Wires.—When a real image is formed by any lens or mirror, it is easy to place a mark (such as

the point of a pen) in coincidence with any selected point of the image. In telescopes, when used for purposes of measurement, there is usually a pair of very fine threads crossing each other, which is thus used as a mark. If the intersection of the threads be made to coincide with a point of the real image formed by the objective, the observer will be able to see both at the same time well defined. When the coincidence is not exact as regards distance from the ocular, not only will there be a difficulty in seeing both distinctly at the same time, but the one will appear to move with respect to the other when he moves his eye sideways. He has thus the means of determining whether the coincidence is exact, and it is his duty to make it exact. When the direction of any object is to be observed, its image is made to coincide with the intersection of the threads; and the line joining the intersection of the threads to the centre of the objective will then, if produced, pass through the object. This joining line is accordingly regarded as the line of sight in making observations of direction by means of a telescope, and it is called the *line of collimation.*

The intersecting threads are called *cross-wires*, and it is to their employment that modern astronomy is mainly indebted for its great superiority to ancient astronomy in the power of accurate observation.

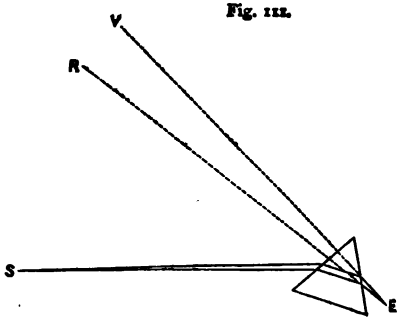
CHROMATIC DISPERSION. SPECTRA.

330. When ordinary light, such as sunlight, gaslight, or candlelight undergoes refraction, it is broken up into different colours. This phenomenon is called chromatic dispersion (χρῶμα, colour).

331. **Solar Spectrum.**—To observe the phenomenon with the utmost simplicity, take a good prism of flint-glass, and look through it at a narrow opening through which direct light from the sky enters a dark room, the prism being held so that its edges are parallel to this opening.

Thus, let S (Fig. 111) be the slit or opening through which the light is admitted, and E the position of the observer's eye, then R V will be the position of the image as seen; but this image, instead of being a line of white light, like that which the slit presents to the naked eye, will be a broad rectangle filled up by a succession of colours, of which red is nearest to the true direction of S, and violet is the most remote. The complete series, as enumerated by Newton, is

Fig. 111.



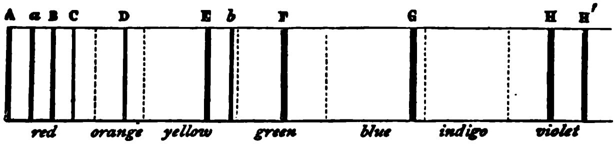
red, orange, yellow, green, blue, indigo, violet.

The appearance thus presented is called the *solar spectrum*. It is due to the fact that the prism has analyzed a beam of solar light into its constituent parts. The image is, in fact, the combination of an indefinite number of different images of the slit, ranged in order and partially overlapping, each of these separate images being similar in shape to the slit itself. Solar light consists of an indefinite number of different constituents, each constituent being distinguished not only by its own proper colour, but also by its own refrangibility. The violet constituents are the most, and the red constituents the least refrangible; in other words, the index of refraction (say from air into a given kind of glass) is not one definite quantity for all the constituents alike, but varies gradually from a minimum for the light which appears at the red end of the spectrum to a maximum for the light which appears at the violet end.

332. **Fixed Lines.**—In the experiment just described,

if the prism is good, the slit sharp, and the observer sufficiently distant from it, the spectrum will be seen to contain several black lines—places where images of the slit are wanting. Whenever these *fixed lines* or *Fraunhofer lines* are seen, they may be taken as an indication that the experiment has been very successfully performed. They indicate great *purity* of the spectrum; in other words, great freedom from overlapping of the different constituents. Fig. 112 shows the positions of the most

Fig. 112.



conspicuous of the fixed lines, and the letters marked above them are the names by which they are known, these names having been assigned to them by their discoverer Fraunhofer.

333. Spectroscope.—A spectroscope,—that is, an instrument for the observation of spectra,—usually contains the following parts:—

1st. A *slit*, the edges of which are two knife-edges of steel very truly ground and exactly parallel. The distance between them—in other words, the width of the slit—is usually adjustable by a screw.

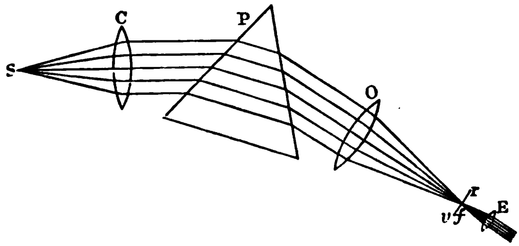
2d. A *collimating lens;* being a convex lens with the slit at its principal focus. The rays from any one point of the slit, therefore, emerge from the lens as a parallel pencil. The slit and lens are let into opposite ends of a tube, whose length is the focal length of the lens. This tube with the lens and slit is called the *collimator*.

3d. A *prism* of dense flint-glass, on which the rays fall after emerging from the collimator. It should be so placed that all the rays are refracted through it with approximately minimum deviation.

All the rays of equal refrangibility from any one point of the slit will emerge from the prism as a parallel pencil.

4th. An *observing telescope*, constructed like an astronomical refractor of small size, and so placed that the rays traverse it after emerging from the prism. It must be focussed as if for viewing a very distant object, because rays of a given refrangibility from a given point of the slit are parallel before entering it. Fig. 113 is a ground

Fig. 113.



plan of the arrangement, showing the course of a pencil of rays of the same refrangibility from a single point of the slit. S is the slit, C the collimating lens, P the prism, O the object-glass of the observing telescope, and E the eye-piece. A real image of the slit will be formed at f by rays of the given refrangibility; other images will be formed between f and v by rays of greater refrangibility, and others between f and r by rays of less refrangibility.

334. Spectrum thrown on Screen.—If, instead of the telescope with its two lenses O, E, we simply employ a convex lens O, we can throw the spectrum upon a screen placed at $v\,r$; but a much more powerful source of light is required for throwing a spectrum on a screen than for viewing it directly. The collimating lens C can be dispensed with, if the distance of the slit from the prism is sufficiently great in comparison with the diameter of the lens O.

335. Spectra of Artificial Lights.—The spectrum obtained is in every case the spectrum of the light which comes through the slit to the prism. The components of this light, whatever they may be, will appear in their proper places in the spectrum.

If the slit is illuminated by a gas or candle flame instead of by solar light, the Fraunhofer lines will not be seen. They are characteristic of solar light exclusively. In other respects the spectrum of a gas or candle flame is very similar to the solar spectrum, being, however, relatively weaker in blue and violet than in red and yellow. The brightest part is, in both cases, in the yellow, from which there is a gradual falling off to both ends.

336. Spectra of Incandescent Solids and Liquids.—Every incandescent solid or liquid (for example, red-hot, white-hot, or molten iron) gives a continuous spectrum. At a dull red heat the spectrum consists mainly of red, and as the temperature rises the spectrum extends further towards violet, so that the red rays, though they increase in absolute brightness, become less predominant. The total light thus becomes whiter. The whiteness of solar as compared with most artificial lights is thus attributable to the high temperature of the source.

337. Bright-line Spectra.—By putting common salt on the wick of a spirit-lamp, we obtain a flame whose spectrum, instead of being a broad band of successive colours, is a narrow line of yellow. The light emitted by this flame is thus proved not to be compounded like daylight of an indefinite number of different constituents, but to be simple—or in technical phrase, *monochromatic*. The simplicity, however, is not absolute. When high spectroscopic power is employed the line is seen to be double, that is to say, it splits up into two lines near together. This line, whether seen as double or single, is called the *sodium line*. It is due to the presence of incandescent vapour of sodium. It is an example of what is called a *bright-line spectrum*. All those metals which are readily vaporized give bright-line spectra when exposed

to the flame of a spirit-lamp, or still better, of a Bunsen burner; and the more refractory metals also give bright-line spectra when vaporized by the electric spark. Gases and vapours usually give bright-line spectra when raised to incandescence. The spectrum of hydrogen, for example, consists of 4 lines, of which one is in the red, two in the blue, and the fourth (which is less bright than the others) in the violet.

338. Hydrocarbons.—All the artificial lights in ordinary use are produced by the combustion of a mixture of hydrogen and carbon. The hydrogen contributes a large amount of heat, but would of itself give very little light. The light actually emitted is believed to be mainly due to incandescent particles of solid carbon,—such particles as are deposited by these flames in the form of lamp-black; and the continuous spectrum which these flames give is believed to be the spectrum of incandescent solid carbon.

339. Ultra-red and Ultra-violet Rays.—The visible solar spectrum is only the middle portion of the whole spectrum of the sun's rays. There are rays of powerful heating effect beyond the red, and rays capable of exerting chemical action beyond the violet. These latter, when allowed to fall upon what are called fluorescent substances, such as canary glass (coloured with oxide of uranium) or solution of quinia sulphate, are converted into visible rays, and give the fluorescent substance the appearance of being self-luminous in its superficial layer.

340. Coincidences of Dark and Bright Lines.—Most of the dark lines in the solar spectrum are found to have the same positions as the bright lines of known terrestrial substances. One of the strongest of them for example—Fraunhofer's line D,—is a double line exactly coincident with the double line of sodium. The coincidence is easily exhibited by holding a salted spirit-flame in front of the slit of a spectroscope directed towards the sky. The observer will see a certain line in the spectrum

become alternately bright and dark as the flame is held in front and removed.

341. Reversal of Lines. Connection between Emission and Absorption.—It is also found that if sodium is burned in such a situation that its vapour—not hot enough to be luminous—is interposed between an artificial light and the slit of a spectroscope, a dark line, having the position of the line D, makes its appearance in the otherwise continuous spectrum. It is, therefore, inferred that the dark lines in the sun's spectrum are due to vapours which surround the brighter body of the sun and intercept certain constituents of the transmitted light, namely, the same constituents which these vapours would emit on their own account if incandescent. Every substance is capable of absorbing the same rays which it is capable of emitting, and carbon, which greedily absorbs all kinds when cold, abundantly emits all kinds when sufficiently heated.

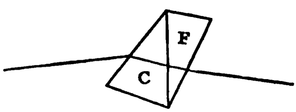
342. Stars and Nebulæ.—The spectroscope has accordingly enabled astronomers to analyze the sun's atmosphere; and the same method has been applied to the stars and nebulæ. Some of the latter give bright-line spectra, while others, which can be resolved by the telescope into clusters of stars, give a continuous spectrum. The spectra of some of the brightest of the stars have been obtained with sufficient distinctness to show the existence in them of dark lines identical with some of those which appear in the solar spectrum.

343. Chromatic Aberration.—The unequal refrangibility of different rays is a source of difficulty in the construction of telescopes and microscopes. Every lens constructed of a single piece of glass has a shorter focus for violet than for red light, and therefore cannot yield a sharp image of a white object. At whatever distance a screen is held to receive the image, it must necessarily be wrong for all the colours but one. Such a lens accordingly gives images bordered by coloured fringes, which are especially noticeable when the object is much brighter

or darker than the back-ground. This defect is called *chromatic aberration.*

344. **Achromatic Prism.**—It is possible, however, by combining two lenses composed of different kinds of glass, to remedy this evil. Two prisms, one of flint and the other of crown glass, if they give the same amount of separation between red and violet rays (an end which can be attained by giving the crown prism a much larger angle than the flint), will not produce equal deviations. The deviation produced by the crown will be greater (for all the colours) than that produced by the flint. Hence, if the two prisms are combined, with their angles turned opposite ways, as in Fig. 114, where C is the crown and F the flint, we obtain a compound prism which will produce deviation (in the direction due to the crown), but does not separate red from violet. Such a prism is called *achromatic.* On the other hand, if two prisms, one of flint and the other of crown, give equal deviations for the mean rays (to which end, the crown prism must still have the greater angle of the two, but the excess must not be so great as before), the separation between red and violet will be greater for the flint than for the crown, and the two combined in the manner of Fig. 114 will produce dispersion without deviation; or, to speak more strictly, will produce deviation of red and violet in opposite directions, so that the spectrum of a slit as seen through it will extend on both sides of the true direction of the slit. Such prisms are used in direct-vision spectroscopes, in which the observer looks in at one end of a straight tube, the slit being at the other end.

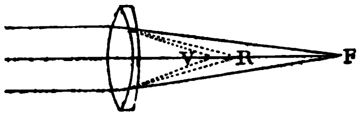
Fig. 114.



345. **Achromatic Lens.**—In like manner, two lenses, one of flint and the other of crown, can be made of such curvatures that the compound lens formed by uniting them, as in Fig. 115, will have the same focal length

for violet as for red. Such compound lenses are called *achromatic*. If the crown lens be convex, the flint lens must be concave, and the combination will have the properties of a convex lens.

Fig. 115.



In the figure the convex lens is of crown; V is its principal focus for violet, and R for red. With respect to the concave lens, which is of flint, F is the focus conjugate to V for violet and to F for red. F is therefore the principal focus of the combination.

346. **Irrationality of Dispersion.**—When we compare carefully two specimens of the solar spectrum obtained by means of two different prisms of different materials (or even of different angles and the same material), we find in general that the distances between the fixed lines are not exactly proportional in the two spectra, —we may find, for example, that red covers a greater space in the first spectrum than in the second, and that blue covers a greater space in the second spectrum than in the first. This fact is called the *irrationality of dispersion*. One consequence of it is that a prism or lens cannot be made perfectly achromatic. A lens composed of *two* pieces can be constructed so that it shall have the same focal length for *two* specified colours, but it will have slightly different focal lengths for the remaining colours.

COLOUR.

347. **Relation of Colours to the Spectrum.**—Unaided vision cannot resolve a beam of light into its constituents, and the same apparent colour may be produced by very various mixtures.

Most of the colours which we see in nature are capable of being imitated by making a mixture of some one colour of the spectrum with white of feeble intensity. Purple is

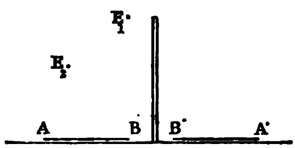
the only exception to this rule; but it can be imitated by a mixture of two colours of the spectrum, with or without white. White itself can be produced by a mixture of two colours of the spectrum, and any two colours which can produce white by their mixture are called *complementary*. Each red constituent of the spectrum is complementary to a constituent lying somewhere in the green; and each orange or yellow is complementary to one of the blues or violets. The yellowish-greens are not complementary to any single constituents of the spectrum, but their complements can be made by mixing red and violet.

348. Mixing of Colours.—All these statements respecting mixtures are to be understood as relating to *mixtures of light*, not to *mixtures of pigments*. The following are some of the best practical methods of obtaining mixtures of given kinds of light.

(1.) Two spectra may be made to overlap, so that at each point there will be a mixture of two colours, one from each spectrum.

(2.) One colour may be seen through a plate of glass, while the other is seen by reflection from the near side of the plate. Thus two pieces of coloured paper A B, A′ B′ (Fig. 116) may be laid on a table, and the glass plate be set upright with its near face midway between them. An image of A B, formed by reflection, will coincide with A′ B′; and an eye, on the same side as A B, will see a mixture of both colours at A′ B′. When the eye is held nearly in the plane of the plate, as at E_1, the reflected image will predominate, because, when light falls on glass nearly tangentially, it is nearly all reflected. By lowering the eye to such a position as E_2, the reflected image will be rendered weak, while the colour seen by transmission will be nearly as strong as if the glass were removed. It is thus easy to vary the ratio in which the colours are mixed.

Fig. 116.



(3.) By arranging the colours as sectors on the face of a disk, and then setting it in rapid rotation.

In methods (2) and (3) the mixture is intermediate in brightness between the constituents. In method (1) it is brighter than either. This method, in fact, gives their sum, and the other two their mean.

349. Three Elementary Colour-sensations.—Quantitative experiments on the mixture of coloured lights have established the conclusion that the eye is only capable of receiving three independent sensations of colour. If one mixture A produces these sensations in the same strength as another mixture B, the two mixtures will appear alike.

Colour-blind persons have only two of the three elementary colour-sensations, and the spectrum appears to them to consist of only two decidedly different colours, with a band of neutral tint between. The extreme red end is invisible to them, and a bright scarlet and a deep green appear to them alike.

There are three colours in the spectrum, and only three, which cannot be imitated by mixtures. One of them is a deep olive-green, and the other two are the extreme red and violet. Hence it is concluded that these three colours come nearer than any others to the three elementary colour-sensations.

The maximum of *intensity* does not, in the case of any of the three sensations, coincide with the maximum of purity, and will, in fact, be different for different specimens of the solar spectrum (art. 346). It will always lie between the position of maximum purity and the brightest part of the spectrum. The two extreme ends of the spectrum excite sensations of great purity but of extremely low intensity.

350. Colour by Transmission.—When solar light comes through a plate of coloured glass or other coloured and transparent material, some of its constituents are largely absorbed (going to heat the substance) while others are transmitted with little loss. The character of

the absorption can in each case be tested by examining the transmitted light with a spectroscope; and it is found in many cases that certain groups of constituents have been absorbed; that is to say, dark bands are seen in the spectrum of the transmitted light. In the case of cobalt-blue glass, the yellow, orange, and scarlet are absorbed, while the extreme red is allowed to pass, forming a deep crimson band, separated from green, blue, and violet by a broad space of darkness. In the case of common red glass (coloured with oxide of copper), all the constituents are almost completely absorbed except those which are red. Hence this glass is often used as a means of obtaining approximately monochromatic light.

351. **Superposition of Coloured Glasses.**—If we select two plates of coloured glass such that the constituents which are transmitted by the one are largely absorbed by the other, the double plate obtained by placing one of them in front of the other will be almost perfectly opaque. This is the case with ordinary red and green glass, which, though they may be highly transparent singly, appear black where they overlap. Such experiments show that the colours of two glasses are not *mixed* by looking through both together. A blue glass and a yellow glass overlapping usually appear green, whereas their colours if mixed would give buff, gray, or white. The mixing of coloured liquids follows the same laws as the superposition of coloured glasses.

352. **Mixtures of Pigments.**—As to mixtures of pigments, whether in the dry or wet condition, all coloured pigments consist of transparent particles, each of which acts like a sheet of coloured glass, reflecting a little light, absorbing another portion, and transmitting a third. They are opaque in bulk, for the same reason that snow is opaque though composed of transparent particles of ice—the light is so broken up by successive reflections in various directions that it cannot penetrate deep. The colour of a mixture of pigments depends—like the colour of overlapping glasses—not merely on the *apparent* colours

of the separate ingredients as seen by the naked eye, but on the composition of these colours as revealed by the spectroscope. If the brightest part of the spectrum of one pigment coincides with a dark band in the spectrum of another, the mixture of the two will be very deficient in brightness as compared with the two pigments separately.

353. **Colours Heightened by Contrast.**—Gazing steadily at any one colour (especially if bright) produces a temporary exhaustion and enfeeblement of the corresponding colour-sensation; so that, if we transfer our gaze to a white wall, the complementary colour-sensation predominates, and the complementary colour is seen. For the same reason, the colours in a picture cannot be appreciated unless they are sufficiently diversified to allow each of the three elementary colour-sensations due amounts of alternate exercise and rest. Complementary colours set each other off to the best advantage.

THE EYE.

354. The eye contains arrangements for throwing images of external objects upon a sensitive membrane called the *retina*. The image thus formed is the origin of the sensation of vision.

The instruments by which the image is formed may be roughly compared to a spherical mass of water, having a double-convex glass lens immersed in it; the whole being inclosed in a thin glass shell. In discussing the formation of images by such a combination, the glass shell may be neglected, since the curvatures of its two faces are sensibly equal and opposite, and we may regard rays from without as directly incident upon the sphere of liquid. Parallel rays thus incident will be rendered convergent, and the convergence will be further increased by the action of the glass lens. It is possible to assign the lens such a position and such curvatures that the rays will come to a focus on the opposite surface of the sphere.

To obtain a well-defined image it would be necessary to employ an annular stop to intercept all circumferential rays and only allow those to pass which are incident near the axis of the lens.

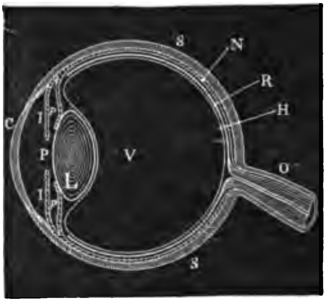
355. In general, a parallel pencil in air, incident nearly normally upon a medium of index μ bounded by a convex surface of radius of curvature r, is brought to a focus at a distance from the surface equal to $\frac{\mu}{\mu - 1} r$. For water, and for the humours of the human eye, μ is approximately $\frac{4}{3}$, therefore the focal length is $4r$. Hence if there were no crystalline lens in the eye, rays entering it from a distant point would converge towards a conjugate focus whose distance from the cornea would be four times its radius of curvature. It is the office of the crystalline lens to diminish this distance and make the conjugate focus fall upon the retina.

The focal length of a given lens varies with the medium in which it is placed, being (as the formula of art. 301 shows) inversely proportional to $\mu - 1$, where μ denotes the index of refraction out of this medium into the lens. Thus, if the index from air to glass be $\frac{3}{2}$, from air to water $\frac{4}{3}$, and therefore from water to glass $\frac{3}{2} \div \frac{4}{3}$, that is $\frac{9}{8}$, the focal length of a glass lens in water will be to its focal length in air as $\frac{1}{2}$ to $\frac{1}{8}$, or as 4 to 1. The index of refraction of the aqueous humour of the eye is 1·337, of the vitreous humour 1·339. That of the crystalline lens varies from about 1·337 in the external layer to 1·400 in the centre. This gradual increase is advantageous as tending to prevent that loss of light by reflection which always accompanies an abrupt change of index. The lens is built up of layers or coats whose forms are not similar one to another, but become more nearly spherical in approaching the centre.

In front of the crystalline lens there is an annular diaphragm called the *iris*. Its aperture, which is called the *pupil*, contracts and enlarges spontaneously according as more or less light falls upon the eye. The interior of the

eye behind the pupil is lined (with the exception of the retina) with an intensely black membrane which serves (like the blackening of the interior of telescopes) to prevent extraneous light from mingling with the light that forms the image.

Fig. 117.



356. In Fig. 117, which represents a section of the human eye, C is the *cornea*, behind which is the anterior chamber, extending as far as the crystalline lens L, and filled with the aqueous humour. I I is the iris, P the pupil, *p p* the *ciliary processes*, which are attached to the lens, V is the *vitreous humour*, R the retina, which is an expansion of the *optic nerve* O. H is the *hyaloid* membrane, N the *choroid*, and S the *sclerotic*.

PHOTOMETRY.

357. Instruments for measuring quantity of light are called *photometers*. The principal purpose to which photometers are applied is the comparison of the quantities of light emitted by different sources.

The illumination of a given plane surface (that is, the light which it receives per unit area) at different distances from a given source (such as a gas or candle flame) varies inversely as the square of the distance, the distance being supposed to be large in comparison with the diameter of the source, and the rays being incident at the same angle in the different cases compared. Thus one candle will produce the same illumination at the distance of one yard as four candles at the distance of two yards. This law is deducible from the axiomatic assumption that if one object exactly screens another from the source, the same

quantity of light which falls upon the nearer would fall upon the more distant if the nearer were removed (see Fig. 107, art. 321).

358. Photometric observations are usually made by employing a translucent screen which is more translucent in one part than in another; for example, a piece of thin paper partly covered with thicker paper. This screen is placed between the two flames with one face towards each, and is moved into such a position that both sides of it are equally illuminated. The illuminating powers of the two flames are then directly as the squares of their distances from it. When such a screen is more strongly illuminated on one side than on the other, an observer on the side which is most strongly illuminated sees the thin part darker than the thick; while to an observer on the less illuminated side the thin part appears the brightest.

VELOCITY OF LIGHT.

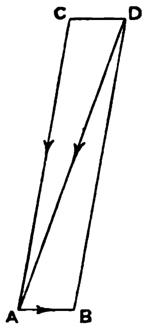
359. Determination by Jupiter's Satellites.—The earliest computation of the velocity of light was based on observations of the eclipses of Jupiter's satellites. On the average of a long series of observations, it is found that these eclipses are visible $16^{m}\ 26^{s}{\cdot}6$ earlier at the point of the earth's orbit nearest to Jupiter than at the diametrically opposite point. Light, therefore, occupies the above time in traversing the diameter of the earth's orbit.

360. By Aberration.—Another determination of the velocity of light is furnished by the astronomical phenomenon called *aberration*.[1] Each star appears to describe in the course of a year a small orbit about its mean place, in such a manner that, if we call the direction in which the earth at any instant is moving the *forward* direction, every star is always on the forward side of its mean place.

[1] The word "aberration" has other senses in optics, but always distinguished by some qualifying epithet, as in the case *matic* aberration" (art. 343). The phrase "aberration of light," without qualification, always denotes the phenomenon here d

This phenomenon is explained in the same way as the familiar fact that a shower of rain falling vertically seems to a person running forwards to be coming in his face. The relative motion of the rain-drops with respect to his body is found by combining the actual velocity of the drops, whether vertical or oblique, with a velocity equal and opposite to that with which he runs. Thus if A B (Fig. 118) represents the velocity with which he runs, and C A the true velocity of the drops, the apparent velocity of the drops will be represented by D A. If a tube pointed along D A moves forward parallel to itself with the velocity A B, a drop entering at its upper end will pass through its whole length without wetting its sides; for while the drop is falling along D B (we suppose with uniform velocity) the tube moves along A B, so that the lower end of the tube reaches B at the same time as the rain-drop. In like manner, if A B is the velocity of the earth, and C A the velocity of light, a telescope must be pointed along A D to see a star which really lies in the direction of A C or B D produced.

Fig. 118.



The angle C A D is called the *aberration of the star.* When the angle B A C is a right angle—in other words, when the star lies in a direction perpendicular to that in which the earth is moving, the aberration amounts (for all stars) to $20''\cdot5$. Hence it is found by computation that the velocity of light is about ten thousand times as great as the velocity of the earth in its orbit. As the earth describes its orbit in a known time, namely, in one year, its velocity can be calculated if we know the size of the orbit, or what amounts to the same thing, if we know the distance of the earth from the sun.

361. **Distance of the Sun.**—Both these methods of determining the velocity of light are therefore dependent on a knowledge of the sun's distance,—an element

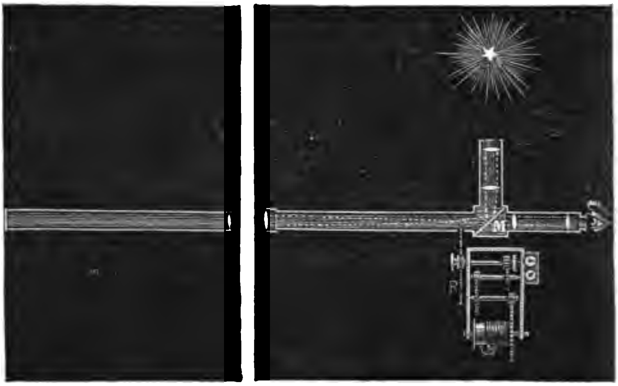
whose accurate determination is a matter of extreme difficulty. Not many years ago it was believed to be 95 or 96 million miles. It is now, on better grounds, believed to be about 92 million miles. This makes the velocity of light about 186,000 miles per second.

362. Fizeau's Method.—In recent years, two methods have been put in practice for determining the velocity of light independently of any knowledge of the distance of the heavenly bodies. In Fizeau's method, a rapidly revolving wheel with square teeth is employed. Each tooth, as it passes, intercepts the light from a powerful source on its way to a distant mirror which reflects it directly back. If, when the light thus reflected has returned to the revolving wheel, it finds one of the teeth in its path, it is stopped and extinguished; if, on the contrary, it finds a space (between two teeth) in its path, it passes through and reaches the observer's eye. It will meet with a space if the distance turned by the wheel in the time of going and returning is equal to one tooth and space, or to any exact multiple of this distance. In the experiments recently executed on this plan by Cornu, the wheel was made to revolve at various measured speeds up to that which caused 21 teeth and spaces to pass; and the distance between the wheel and the mirror was 23 kilometres (a kilometre being about $\frac{5}{8}$ of a mile.

363. Fizeau's Apparatus.—Fig. 119 will give an idea of the apparatus employed. R is the toothed wheel, M is a sheet of glass fixed at an angle of 45° to the plane of the wheel. The light of an electric lamp shining in through a lateral tube containing two lenses falls upon the sheet of glass, and is reflected by it in the direction of the distant station, the arrangements being such that an image of the lamp-flame is formed precisely in the plane of the wheel and coinciding (as the wheel revolves) alternately with a tooth and a space. This image is in the principal focus of a large convex lens (the objective of a telescope), from which the rays emerge in parallel pencils and travel to the distant station, where they pass through

another large convex lens and form another image at its principal focus. A plane mirror, exactly coinciding with this image, reflects them back by the same path by which

Fig. 119.



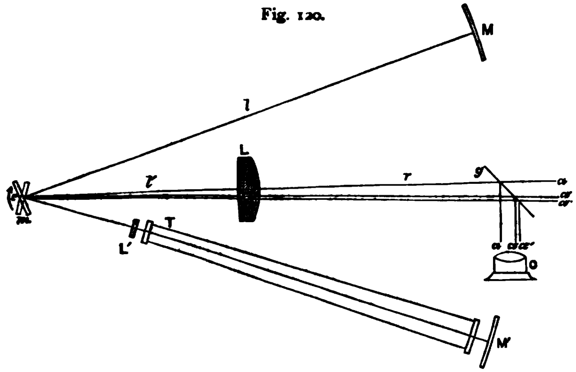
they came; and if there is no tooth in the way, they pass through the glass sheet M, and enter the eye of the observer through the eye-piece of the telescope.

It is of course to be understood that the light which completes the path thus indicated is only a small part of the original light. A large portion goes astray in the passage from one station to the other; and other portions are lost by imperfect reflection and imperfect transmission.

364. **Foucault's Experiment.**—Another method was successfully put in practice by Foucault, and has since been improved by Michelson. A beam of bright sunshine reflected from an external mirror enters a room by a square hole at *a* (Fig. 120), which has a fine platinum wire stretched across it to serve as a mark. The beam is concentrated by an achromatic lens L, and before coming to a focus falls upon a plane mirror *m* revolving about an axis at *c* in its own plane, which is perpendicular

to the plane of the figure. In one part of the revolution the reflected beam is directed upon a concave mirror M, whose centre of curvature is at *c*, so that the beam is reflected back to the revolving mirror and thence back through the lens to the hole at which it first entered. The distance between the two mirrors *m* and M is such that the real image of the platinum wire exactly coincides with M, and hence, by the property of conjugate foci, a

Fig. 120.



second real image would be formed at *a* in coincidence with the platinum wire itself, if the mirror *m* had not shifted its position between the first and the second incidence of the ray upon it. But in consequence of the rapid rotation of the mirror, the change in its position is sufficient to make the angle of incidence of the returning ray appreciably different from that of the original ray, and the image is accordingly formed not at *a*, but at a neighbouring point *a*'.

A sheet of glass *g* placed at an angle of 45° reflects a portion of the returning light, and causes it to form an image at *α* or at *α*', according as the mirror *m* is at rest or revolving. This image is in the focus of the eyepiece O, through which the observer looks. The dis-

tance between a and a'—that is, the displacement of the image by the rotation of the mirror—is carefully measured, and furnishes the means of determining the angle turned by the revolving mirror during the time that light travels from m to M and back. As the speed of rotation of the mirror is known, the angle turned gives a measure of the time occupied, and in this time light has travelled a measured distance, hence the velocity of light is determined. a'' represents the position of another displaced image formed at the same time by rays that have traversed the tube T, which is filled with water, M′ being a second concave mirror at the same distance from c as the other mirror M. The displacement of the water-image a'' was greater than that of the air-image a', hence the time occupied by light in going from c to M′ and back was greater than the time from c to M and back. Thus it was for the first time proved that light travels slower in water than in air, the ratio of the two velocities, as nearly as it could be determined, being 4 to 3. This is in accordance with the undulatory theory of light, which asserts that the index of refraction from one medium into another is equal to the ratio of the velocity in the first medium to the velocity in the second.

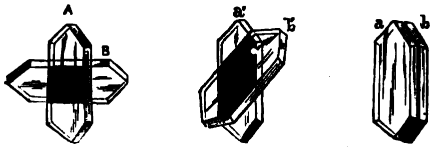
Michelson's determination of the velocity of light in air was 299·7 million metres per second and Cornu's 300·3. The velocity then is very approximately 300 million metres or 3×10^{10} centimetres per second. This is about 186,400 miles per second.

POLARIZED LIGHT.

365. Polarization by Tourmaline.—When a piece of the semi-transparent mineral called tourmaline is cut into slices by sections parallel to its axis, it is found that two of these slices, if laid one upon the other in a particular relative position, as A, B (Fig. 121), form an opaque combination. Let one of them, in fact, be turned round upon the other through various angles (Fig. 121). It will

be found that the combination is most transparent in two positions differing by 180°, one of them *ab* being the natural position which they originally occupied in the crystal; and that it is most opaque in the two positions

Fig. 121.



at right angles to these. It is not necessary that the slices should be cut from the same crystal. Any two plates of tourmaline with their faces parallel to the axes of the crystals from which they were cut, will exhibit the same phenomenon. The experiment shows that light which has passed through one such plate is in a peculiar and so to speak unsymmetrical condition. It is said to be *plane-polarized.* According to the undulatory theory, a ray of common light contains vibrations in all planes passing through the ray, and a ray of plane-polarized light contains vibrations in one plane only. Polarized light presents the same appearance as common light to the naked eye; and in experiments in polarization two pieces of apparatus must be employed—one to produce polarization, and the other to show it. The former is called the *polarizer*, the latter the *analyzer;* and every apparatus that serves for one of these purposes will also serve for the other. In the experiment above described, the plate next the eye is the analyzer. The usual process in examining light with a view to test whether it is polarized, consists in looking at it through an analyzer, and observing whether any change of brightness occurs as the analyzer is rotated. When the light of the blue sky is thus examined, a difference of brightness can always be detected according to the position of the analyzer, especially at the

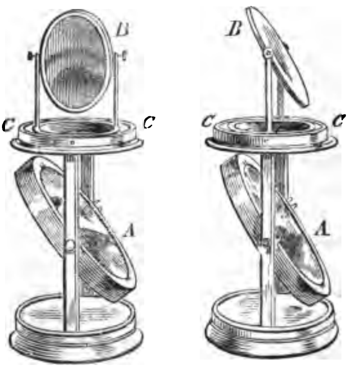
distance of about 90° from the sun. In all such cases there are two positions, differing by 180°, which give a minimum of light, and the two positions intermediate between these give a maximum of light.

The extent of the changes thus observed is a measure of the completeness of the polarization of the light.

366. Polarization by Reflection.—Transmission through tourmaline is only one of several ways in which light can be polarized. When a beam of light is reflected from a polished surface of glass, wood, ivory, leather, or any other non-metallic substance, at an angle of from 50° to 60° with the normal, it is more or less polarized, and in like manner a reflector composed of any of these substances may be employed as an analyzer. In so using it, it should be rotated about an axis parallel to the incident rays which are to be tested, and the observation consists in noting whether this rotation produces changes in the amount of reflected light.

Malus' Polariscope (Fig. 122) consists of two reflectors A, B, one serving as polarizer and the other as analyzer, each consisting of a pile of glass plates. Each of these reflectors can be turned about a horizontal axis; and the upper one (which is the analyzer) can also be turned about a vertical axis, the amount of rotation being measured on the horizontal circle C C. To obtain the most powerful effects, each of the reflectors should be set at an angle of about 33° to the vertical, and a strong beam of com-

Fig. 122.



mon light should be allowed to fall upon the lower pile in such a direction as to be reflected vertically upwards. It will thus fall upon the centre of the upper pile, and the angles of incidence and reflection on both the piles will be about 57°. The observer looking into the upper pile, in such a direction as to receive the reflected beam, will find that, as the upper pile is rotated about a vertical axis, there are two positions (differing by 180°) in which he sees a black spot in the centre of the field of view, these being the positions in which the upper pile refuses to reflect the light reflected to it from the lower pile. They are 90° on either side of the position in which the two piles are parallel; this latter, and the position differing from it by 180°, being those which give a maximum of reflected light.

367. Polarizing Angle.—For every (non-metallic) reflecting substance there is a particular angle of incidence which gives complete polarization in the reflected light. It is called the *polarizing angle* for the substance in question, and is that angle of incidence for which the reflected and refracted rays are at right angles.

368. Double Refraction.—Many crystals possess the property that rays refracted into them are split up into two parts which take different directions, so that

Fig. 123.

objects looked at through such crystals often appear double, as in Fig. 123, which represents printed words as

seen through a rhomb of Iceland-spar. This phenomenon is called *double refraction.* In every case the two rays obtained by double refraction are completely polarized, and their planes of polarization are at right angles to each other.

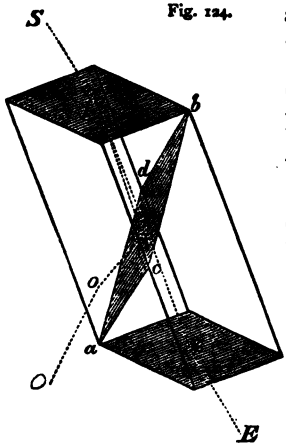


Fig. 124.

369. Nicol's Prism.—One of the most efficient polarizers and analyzers is Nicol's prism, which is made by slitting a rhomb of Iceland-spar along a diagonal plane *abcd* (Fig. 124), and cementing the two pieces together with Canada balsam. The critical angle of total reflection (art. 267) for a ray passing out of the crystal into the balsam, depends on the relative index of refraction from one of these media to the other. This index is different for the two rays into which the incident ray S I is split up by the double refraction of the crystal; hence an angle of incidence, which is small enough for the transmission of one of them, may cause total reflection of the other. The courses taken by the two rays are indicated in the figure. One of them is totally reflected on meeting the Canada balsam, and passes out of the crystal on one side; the other ray is refracted through the balsam, and finally emerges at the opposite end of the prism from that at which it entered.

370. Colours produced by Polarization.—There are several ways in which beautiful displays of colour can be obtained by means of polarized light,—usually by introducing a thin slice of a crystal between the polarizer and analyzer. The explanation of these phenomena is too intricate for an elementary treatise.

EXAMPLES IN LIGHT.

XXVIII.—Reflection by Plane Mirrors. Arts. 266–272.

1. Two parallel plane mirrors face each other at a distance of 3 feet, and a small object is placed between them at a distance of 1 foot from the first mirror, and therefore of 2 feet from the second. Calculate the distances, from the first mirror, of the three nearest images which are seen in it; and make a similar calculation for the second mirror.

2. Two plane mirrors contain an angle of 120°, and a small object is at a distance of 40° from one of them. Draw a figure showing the positions of the two images which are formed, and state their distances (in degrees) from this mirror.

3. Show that a person standing upright in front of a vertical plane mirror will just be able to see his feet in it, if the top of the mirror is on a level with his eyes, and its height from top to bottom is half the height of his eyes above his feet.

4. A square plane mirror hangs exactly in the centre of one of the walls of a cubical room. What must be the size of the mirror that an observer with his eyes exactly in the centre of the room may just be able to see the whole of the opposite wall reflected in it except the part concealed by his body.

XXIX.—Refraction. Arts. 276–293.

1. Calculate the index of refraction of a substance in which the critical angle is 45°.

2. Calculate the index of refraction of a substance in which the critical angle is 30°

3. Show that a ray incident at 45° in glass will be totally reflected, but that a ray incident at 45° in water will be refracted into the air.

4. If the index of refraction from air into glass be $1\frac{1}{2}$, and from glass into diamond $1\frac{2}{3}$, find the index of refraction from air into diamond.

5. If the index of refraction from air into crown-glass be $1\frac{1}{2}$, and from air into flint-glass $1\frac{3}{5}$, find the index of refraction from crown-glass into flint-glass.

6. The index of refraction from water into oil of turpentine is 1·11; find the index of refraction from air into oil of turpentine.

7. The index of refraction for a certain glass prism is 1·6, and the angle of the prism is 10°. Find approximately the deviation of a ray refracted through it nearly symmetrically.

8. A speck in the interior of a piece of plate glass (index $1\frac{1}{2}$) appears to an observer looking normally into the glass to be $\frac{1}{4}$ of an inch from the near surface. What is its real distance?

9. The rays of a vertical sun are brought to a focus by a lens at a distance of 1 foot from the lens. If the lens is held just above a smooth and deep pool of water, at what depth in the water will the rays come to a focus?

XXX.—Spherical Mirrors. Arts. 294–302.

1. Find the focal length of a concave mirror whose radius of curvature is 2 ft., and find the position of the image of a point 15 inches in front of the mirror.

2. Find the position of the image of a point 10 ft. in front of the same mirror.

3. Find the position of the image of a point 9 in. in front of the same mirror.

4. Find the position of the image of a point 1 in. in front of the same mirror.

5. Calling the diameter of the object unity, find the diameters of the image when the distances of the object from the mirror are those stated in questions 1, 2, 3, 4.

XXXI.—Lenses. Arts. 311–323.

1. Compare the focal lengths of two lenses of the same size and shape, one of glass and the other of diamond, their indices of refraction being respectively 1·6 and 2·6.

2. If the index of refraction of glass be $\frac{3}{2}$, show that the focal length of an equi-convex glass lens is the same as the radius of curvature of either face.

3. The focal length of a convex lens is 1 ft. Find the positions of the image of a small object when the distances of the object from the lens are respectively 20 ft., 2 ft., and $1\frac{1}{2}$ ft. Are the images real or virtual?

4. When the distances of the object from the lens in last question are respectively 11 in., 10 in., and 1 in., find the distances of the image. Are the images real or virtual?

5. Calling the diameter of the object unity, find the diameter of the image in the six cases of questions 3, 4, taken in order.

6. Show that, when the distance of an object from a convex lens is double the focal length, the image is at the same distance on the other side.

7. The object is 6 ft. on one side of a lens, and the image is 1 ft. on the other side. What is the focal length of the lens?

8. The object is 3 in. from a lens, and its image is 18 in. from the lens on the same side. Is the lens convex or concave, and what is its focal length?

9. The object is 12 ft. from a lens, and the image 1 ft. from the lens on the same side. Find the focal length, and determine whether the lens is convex or concave.

10. A person who sees best at the distance of 3 ft., employs convex spectacles with a focal length of 1 ft.

At what distance should he hold a book, to read it with the aid of these spectacles?

11. A person reads a book at the distance of 1 ft. with the aid of concave spectacles of 6 in. focal length. At what distance is the image which he sees?

XXXII.—TELESCOPES. Arts. 305–310 and 324–326.

1. The objective of a telescope has a focal length of 20 ft. What will be the magnifying power when an eye-piece of half-inch focus is used?

2. Show that if the two specula of a Gregorian telescope have the same focal lengths as those of a Cassegranian telescope, the distance between the two specula is greater in the Gregorian than in the Cassegranian by twice the focal length of the smaller speculum.

3. Show that if the two lenses of an astronomical refractor have the same focal lengths as those of a Galilean telescope, the Galilean telescope will be shorter than the other by twice the focal length of the eye lens.

XXXIII.—PHOTOMETRY. Arts. 357, 358.

1. A lamp and a standard candle give equal illuminations to a screen which is 3 ft. 4 in. from the latter and 10 ft. from the former. How many candle power is the lamp?

2. If a gas flame is 16 candle power, and is 7 ft. 6 in. distant from a standard candle, at what intervening point must the screen be placed to be equally illuminated by both?

ANSWERS.

XXVIII.—Ex. 1. 1, 5, and 7 ft. behind first mirror; 2, 4, and 8 ft. behind second. Ex. 2. 40°, 80°, and 160°. Ex. 4. Side of mirror must be $\frac{1}{3}$ of edge of cube.

XXIX.—Ex. 1. $\sqrt{2}$. Ex. 2. 2. Ex. 3. Because $1\frac{1}{2}$ is greater and $1\frac{1}{3}$ is less than $\sqrt{2}$. Ex. 4. $2\frac{1}{2}$. Ex. 5. $\frac{16}{15}$. Ex. 6. 1·48. Ex. 7. 6°. Ex. 8. $\frac{3}{8}$ in. Ex. 9. 1 ft. 4 in.

XXX.—Ex. 1. Focal length 1 ft.; image 5 ft. in front of mirror. Ex. 2. $1\frac{1}{9}$ ft. in front of mirror. Ex. 3. 3 ft. behind mirror. Ex. 4. $1\frac{1}{11}$ in. behind mirror. Ex. 5. 4, $\frac{1}{9}$, 4, $1\frac{1}{11}$.

XXXI.—Ex. 1. Focal length of diamond lens is $\frac{3}{8}$ of focal length of glass lens. Ex. 3. $1\frac{1}{19}$ ft., 2 ft., 3 ft. on other side of lens. All real. Ex. 4. 11 ft., 5 ft., $1\frac{1}{11}$ in. on same side of lens. All virtual. Ex. 5. $\frac{1}{19}$, 1, 2, 12, 6, $1\frac{1}{11}$. Ex. 6. $1\frac{1}{11}$. Ex. 7. $\frac{6}{7}$ ft. Ex. 8. $3\frac{3}{8}$ in., convex. Ex. 9. $1\frac{1}{11}$ ft., concave. Ex. 10. 9 in. Ex. 11. 4 in.

XXXII.—Ex. 1. 480 diameters.

XXXIII.—Ex. 1. 9 candles. Ex. 2. 1 ft. 6 in. from candle.

SOUND.

371. It is a matter of every-day observation that sound is connected with vibration. A bell, a tumbler, or a tuning-fork, when emitting a musical note, is instantly rendered silent by damping its vibrations with the finger. Every sound has its origin in the vibrations of the body which emits it.

PROPAGATION OF SOUND.

372. The propagation of sound from the sonorous body to the ear is usually effected by means of the intervening air; and if a bell suspended by a string of loose texture is struck in the vacuum furnished by an ordinary air-pump, scarcely any sound is heard.

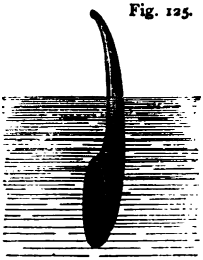
Other elastic bodies as well as air and gases are, however, capable of propagating sound. The sound of a bell struck under water has been propagated through the water to a distance of several miles, and an ear applied to one end of a log of timber (if not rotten) hears the scratch of a pin on the other end. Such bodies as cloth, rotten wood, ashes, or cotton wool, on the other hand, destroy sound very quickly. The transmission of sound is effected by means of vibrations of successive portions of the medium through which it travels, and when sound is extinguished there is probably a production of heat equivalent in amount to the sonorous energy destroyed.

373. **Velocity of Sound.**—The velocity of sound in air has been repeatedly measured by observing the interval which elapses between seeing the flash of a gun fired at night at a distance of several miles and hearing the report. This velocity varies with the temperature—

being greatest when the air is warmest—but is independent of the barometric pressure. At the mean temperature of this country (say 10° C. or 50° F.) it is about 1100 feet or 340 metres per second.

374. **Velocity in Water.**—The velocity of sound in water was measured in 1826 at the Lake of Geneva. Two boats were moored at a distance of 13,500 metres (between 8 and 9 miles). One of them carried a bell weighing about 140 lbs., immersed in the lake. Its hammer was moved by an external lever, so arranged as to ignite a small quantity of gunpowder at the instant of striking the bell. An observer in the other boat was enabled to hear the sound by applying his ear to the extremity of a trumpet-shaped tube (Fig. 125), having its lower end covered with a membrane, and facing towards the direction from which the sound proceeded. By noting the interval between seeing the flash and hearing the sound, the velocity was found to be 1435 metres per second, or more than four times the velocity in air.

Fig. 125.



375. **Pulses.**—The mechanism by which sound is propagated through the air may be thus described. If the air at one point or over one surface is suddenly compressed or expanded, its pressure is thereby rendered either greater or less than that of the air around. The contiguous air is accordingly pushed either from or towards the source. This movement destroys the equilibrium of the next layer of air, which in its turn is pushed from its place, thus producing a similar displacement of the next, and so on. What is called a *pulse* or *wave* is thus propagated through the air, and it is very important for the student in commencing the study of acoustics to form a clear idea of what is meant by a pulse.

376. One of the best ways of exhibiting a pulse is to

suspend a long indian-rubber cord in such a position that it is straight or nearly so (it may be either horizontal, vertical, or inclined), and to strike it sharply near one end. A pulse will be seen to run along the cord to the other end. A pulse can also be made to run along a rope lying on the ground by lifting one end with the hand and giving it a sudden throw without letting go of it.

When such pulses as these travel along a cord, each point of the cord moves to one side and back again while the pulse is passing it. Pulses which are propagated through air differ from these inasmuch as the particles of air through which a pulse passes do not move sideways, but forwards and backwards; in other words, their motion is not perpendicular but parallel to the line along which the pulse travels. Their velocity even when at its maximum is extremely small compared with the velocity of the pulse; and the distances that they are moved by the pulse is extremely small, whereas the distance that the pulse travels may be very great. Although the propagation of a pulse implies the movement of the matter or medium through which it travels, its velocity must therefore not be confounded with the velocity of the particles of this medium. The velocity of a pulse is not the velocity with which matter moves, but is the velocity with which a *form* or *change of condition* moves through matter.

377. Undulation. Waves.—A series of pulses travelling through the same portion of matter in regular succession constitute an *undulation*, and if an undulation can be divided into successive parts which are alike or nearly alike, these successive parts are called *waves*. A pulse may be either a wave or part of a wave; for example, a wave may and often does consist of a pulse of compression followed by a pulse of extension.

378. Period. Wave-length. Phase.—When a series of waves all precisely alike are traversing a medium, each particle of the medium is kept in a continual state of *vibration*—that is, of movement to and fro, or round and round—and the successive vibrations of any one particle

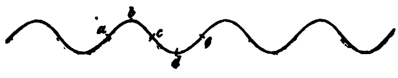
are all precisely alike. The time that a particle takes to go through one complete cycle of motions is called the *periodic time*, or simply the *period*, and is the same for all the particles. Every particle is in the same condition at the end of a period as at its beginning.

The length of a wave, or the *wave-length* of a series of similar waves, is the distance, measured in the direction in which the waves are travelling, between any particle and the next particle that is in precisely the same condition, or, to use technical language, in the same *phase*. Thus, if a number of similar waves are traversing a sheet of water, the distance, in a straight line, from the crest of one wave to the crest of the next is a wave-length.

379. Relation between Velocity, Wave-length, and Period.—The number of waves that pass a particle in a given time—say in one second—is the same as the number of complete vibrations made by the particle in that time. Hence during one period—the time of one complete vibration—the waves advance a distance of one wave-length; and the *velocity of the undulation*, or the distance advanced by the waves in the unit of time, will be the *product of the wave-length by the number of vibrations in the unit of time.*

380. Simple Waves.—When waves are of such a character as to produce the impression of a single musical tone unmixed with others, each wave consists of four parts, which are reversed copies of each other, like *a b*, *b c*, *c d*, *d e* in the annexed figure (Fig. 126). *b* may denote a point

Fig. 126.



of maximum density, *d* a point of minimum density, and *a* and *c* two points at the natural or undisturbed density. The distance (measured in a straight line in the direction in which the waves travel) from a point of maximum or

minimum density to a point of undisturbed density is therefore a quarter of a wave-length.

381. In a medium traversed by such a system of waves, a complete vibration of any particle consists (like a complete vibration of an ordinary pendulum) of four parts, which are reversed copies of each other.

In the case of waves on the surface of a sheet of water, each particle moves in an ellipse lying in a vertical plane; and if A A′, B B′ be the two axes of the ellipse, the four movements are (1) from A to B, (2) from B to A′, (3) from A′ to B′, (4) from B′ to A.

In the case of sound-waves travelling through air the four parts are (1) from the middle to one extreme, (2) from this extreme back to the middle, (3) from the middle to the other extreme, (4) from this extreme back to the middle.

382. The movement in this case precisely resembles the movement of any point of an ordinary pendulum vibrating through so small an angle that the path of each particle is sensibly a straight line; and as in the case of such a pendulum, so also for the vibrating particle, the periodic time is independent of the *amplitude*, that is of the *distance of the extreme positions from the middle position*. If the sound is produced in the open air, and is more loudly heard near than at a distance, the amplitude will be greater near than at a distance, but the periodic time will be the same.

REFLECTION OF SOUND.

383. There are two opposite ways in which sound can be reflected.

If a pulse of compression travelling through a tube meets a solid obstacle which closes the end of the tube, it is sent back upon its course with perhaps a loss of intensity, but, upon the whole, in the same shape in which it arrived—that is, as a pulse of compression; and in like manner a pulse of extension is reflected as a pulse of extension. Such reflection may be called *positive*.

On the other hand, if a tube is open at the end, a pulse travelling through the tube, when it arrives at this end is reflected in the opposite shape to that in which it arrived. For instance, a pulse of compression bursts out with so much vigour that it produces extension behind it, and a pulse of extension is thus originated which travels back through the tube. A compression is thus reflected in the shape of an extension, and an extension in like manner is reflected in the shape of a compression. Such reflection may conveniently be called *negative*. Negative reflection also occurs in the case of transverse pulses travelling along a string, and reflected from the end, when the latter is attached to a solid support. This can easily be exhibited to the eye by the experiment already described with the indian-rubber cord (art. *376*). If, for instance, the cord be horizontal, a pulse of depression, produced by striking the cord sharply on its upper side, will, on arriving at the end, be reflected in the shape of a pulse of elevation—a protuberance on the opposite side of the cord to the original protuberance.

384. **Direction of Reflection.**—The direction of reflected sound follows the same law as the direction of reflected light. That is to say, when sound-waves meet a reflecting surface, the normal to this surface at any point of incidence bisects the angle between the directions of the incident and the reflected waves. This can be illustrated by hanging a watch in the focus of a parabolic reflector. Its ticking will be audible at a great distance directly in front, but not in other directions.

385. **Echo.**—Echo is a familiar illustration of the reflection of sound; and the time which elapses between uttering a shout and hearing its echo is the time required by sound to travel over double the distance from the shouter to the reflecting body.

RESONANCE.

386. An inclosed or partially-inclosed body of air has one or more tones to which it naturally resounds. For

example, a tube open at one end and closed at the other, resounds (that is, the air within it resounds) to a tone whose wave-length in air is about four times the length of the tube. If a tuning-fork producing this tone be held opposite the open end, the resonance of the tube will greatly strengthen the sound. This is explained by the circumstance that a pulse travelling through the tube and reflected from the ends will have run over four times the length of the tube before its cycle of changes is completed. A pulse propagated from the open to the closed end is there positively reflected, and travels back to the open end in its original shape. On arriving at the open end it is negatively reflected (art. 383), and in its reversed shape again travels over twice the length of the tube. On its next arrival at the open end a second negative reflection restores it to its original shape.

A single pulse once started is competent to produce in this way a long series of reflected pulses, each a little weaker than its predecessor; and on each arrival at the open end a pulse is emitted into the surrounding air. The tuning-fork originates a series of pulses which keep time with these arrivals. The successive reflections of a number of original pulses are thus blended together, and the waves emitted from the mouth of the tube have consequently very much greater amplitude than those produced by the unaided fork.

On the same principle, a heavy body hung by a cord can be set swinging through a large arc by a succession of feeble impulses so timed as always to help it on its way. This principle is one of very extensive application in acoustics as well as in other parts of physics. Any mode of vibration which a body can keep up of itself when once started is called a *natural mode of vibration* of the body; and every such mode of vibration is easily excited even by very feeble external action, if regularly applied to the body at intervals precisely equal to the periodic time.

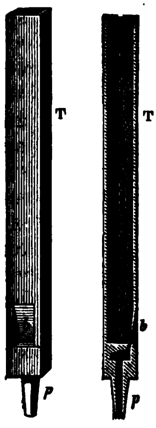
387. Tube open at Both Ends.—For the body of air within a tube open at one end and stopped at the

other, the periodic time is, as we have seen, approximately equal to the time in which a pulse would travel in air through a distance equal to four times the length of the pipe.

In a tube open at both ends, a pulse propagated from one end to the other is there negatively reflected, and returns to the first end in reversed shape. It is there again negatively reflected and restored to its original shape. The whole distance travelled in the periodic time is therefore twice the length of the tube. The periodic time for a tube open at both ends is therefore only half the periodic time for a tube of the same length open at one end and stopped at the other.

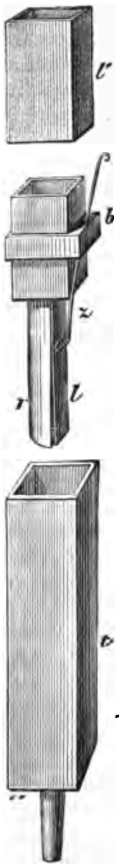
388. Open Organ-pipes.—In all wind-instruments except those which belong to the *reed* class (art. 390), the pitch of the sound emitted depends upon the dimensions and form of a vibrating column of air. Figs. 127, 128 represent the commonest form of organ-pipe. The wind enters at *p* and passes into a small chamber, from which it issues through a narrow opening or slit, which is nearly opposite the sharp edge of the wedge-shaped piece *b*, called the *lip*. The wind issuing from the slit grazes the lip, and would produce a mere puffing sound of no musical character were it not for the resonance of the column of air in the body of the pipe T. The puffing sound to which we have alluded is in fact a confused mixture of a number of faint sounds, among which the tone to which the column of air resounds is one. The tone actually produced is accordingly that to which the column of air resounds. Its wave-length is rather more than double the length of the body of the pipe.

Fig. 127. Fig. 128.



389. Stopped Pipe.—Pipes of the kind just described are called open. A *stopped* organ-pipe differs from one of these in having an air-tight piston, or stopper, at its upper end; and the tuning of such pipes is effected by pushing the stopper further in or drawing it further out. It gives a tone whose wave-length is rather more than four times the length of the body of the pipe, or twice as great as that of the same pipe when unstopped.

Fig. 129.



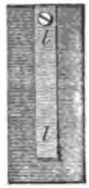
390. Reed Pipe.—Fig. 129 represents a *reed* organ-pipe, with its different parts separated. The wind enters at the lower end, and escapes from the body of the pipe *t* through the reed *r*. *l* is a thin metallic spring called the *tongue*, which, when no wind is passing, leaves a narrow opening between itself and the edges of the tube *r*. When the pipe is blown, the tongue vibrates, and the passage for the wind is alternately opened and closed. The air, being thus compelled to escape in bursts, is thrown into forced vibration, the period of which is the same as that of the vibrations of the tongue. The tongue, however, does not vibrate exactly in its own natural period, but is influenced by the natural resonance of the body of air in the pipe. *z* is a sliding wire for tuning the pipe by lengthening or shortening the portion of the tongue which is free to vibrate.

To obtain a good note, there ought to be but little difference between the natural period of the vibrating tongue and that of the resonant air. The shape of the tongue is also very material. It ought to come in contact with the edges of the tube by rolling itself down

upon them, and not by striking along its whole length at once.

391. **Free Reed.**—The above is the kind of reed chiefly used for organs. In the harmonium and concertina the sound is produced by free reeds, one of which is represented in Fig. 130. It is called *free* because the tongue *ll* has room to pass completely through the opening which it covers, and vibrates backwards and forwards through this opening without striking the edges.

Fig. 130.



PITCH, AND MUSICAL INTERVALS.

392. **Pitch.**—Regularity of recurrence is the characteristic which distinguishes musical sounds from unmusical noises, and the number of recurrences in a given time determines the pitch of the sound. If two vibrating bodies make the same number of vibrations per second, the sounds emitted by them will have the same pitch, however much they may differ in other respects.

The normal human voice produces from about 80 to about 800 vibrations per second; the former number corresponding to a very deep bass, the latter to a very high treble.

393. **Interval of the Octave.**—When two notes differ in pitch by an *octave*, the upper one has exactly twice as many vibrations as the lower. Thus, the middle C of the bass has 128 vibrations per second, the tenor C 256, and the treble C, or an ordinary C tuning-fork, 512.[1]

394. **Consonance and Dissonance.**—Two notes of the same pitch when sounded together are said to be in *unison*, and two notes of different pitch when sounded

[1] Some musicians adopt a higher standard of pitch than others. The treble C varies from about 500 up to about 546, according to the standard adopted. We have adopted in the text what is sometimes called the *theoretical* pitch. It is that adopted by the best makers of acoustic apparatus, and is defined by the rule that every power of 2 is the number of vibrations per second of one of the octaves of C.

together are either *consonant* or *dissonant*. The former name is applied when the combination produces a smooth and pleasing effect, the latter when the effect is jarring and displeasing. The epithets "consonant" and "dissonant" are often applied to the interval between the two notes as well as to the notes themselves. The octave is a very perfect consonance; and a small deviation from exactness converts it into a very offensive dissonance. This interval can therefore be judged by the ear with great precision.

395. **Interval of the Fifth.**—Next in importance to the octave among musical intervals may be reckoned the *fifth*. It is the interval between two notes whose vibrations (in a given time) are as 2 to 3. It forms (like the octave) a very smooth combination, and one which a small error suffices to convert into decided dissonance.

396. **Gamut.**—The notes of the major scale, when not altered and accommodated to suit the imperfections of keyed instruments, are represented by the following series of numbers,—

24, 27, 30, 32, 36, 40, 45, 48

That is to say, the numbers of their respective vibrations in a second (or other fixed time) are proportional to the numbers in this list.

The names *major second*, *major third*, *major fourth*, &c., are given by musicians to the intervals from the first note of this scale to the second, third, fourth, &c. Thus the major second is expressed by the ratio $\frac{27}{24}$, or in lowest terms $\frac{9}{8}$, the major third by $\frac{30}{24}$ or $\frac{5}{4}$, the major fourth by $\frac{32}{24}$ or $\frac{4}{3}$, the major fifth by $\frac{36}{24}$ or $\frac{3}{2}$, the major sixth by $\frac{40}{24}$ or $\frac{5}{3}$, the major seventh by $\frac{45}{24}$ or $\frac{15}{8}$, and the octave by $\frac{48}{24}$ or $\frac{2}{1}$.

If we calculate the intervals between the successive notes of the scale, we find them (in lowest terms) to stand as follows:—

$$\frac{9}{8},\quad \frac{10}{9},\quad \frac{16}{15},\quad \frac{9}{8},\quad \frac{10}{9},\quad \frac{9}{8},\quad \frac{16}{15}.$$

397. **Temperament.**—The first or lowest note of the above scale (represented by 24) is called the *key-note*, and the same name is given to every note which is an octave or any exact number of octaves above or below it. The scale in fact repeats itself over and over again (like the figures of a circulating decimal), so that notes which differ by an octave, or any exact number of octaves, are similarly related to their respective neighbouring notes, and are often spoken of as identical.

As different pieces of music have not all the same key-note, it is necessary to provide means for shifting the above scale (or something like it) up and down the range of keys on a piano or organ. This cannot conveniently be done without a sacrifice of exactness, called *temperament*, and it is customary in the present day to employ a system of tuning (called *equal temperament*), which favours no one key-note above another. The interval from the key-note (24) to its octave (48), instead of being divided, as above, into 7 parts having 3 different lengths ($\frac{9}{8}$, $\frac{10}{9}$, and $\frac{16}{15}$), is divided into 12 parts, as nearly equal as the tuner can make them. Each of these parts is called a *semitone*, and two of them together make a *tone*, in the sense in which this word is used by practical musicians. The series of intervals

tone, tone, semitone, tone, tone, tone, semitone

is employed instead of the series

$$\frac{9}{8},\ \frac{10}{9},\ \frac{16}{15},\ \frac{9}{8},\ \frac{10}{9},\ \frac{9}{8},\ \frac{16}{15}.$$

The *tone* of this system is intermediate between $\frac{9}{8}$ and $\frac{10}{9}$, but much nearer to the former. The *semitone* is considerably less than $\frac{16}{15}$.

Equal intervals are those which are denoted by equal ratios. Hence to find what interval is double of the interval $\frac{16}{15}$ we must square the fraction $\frac{16}{15}$; this gives $\frac{256}{225}$, or $1\frac{31}{225}$, which is intermediate between $1\frac{1}{7}$ and $1\frac{1}{8}$. Hence the interval $\frac{16}{15}$ is more than half the interval $\frac{9}{8}$.

398. **Methods of Counting Vibrations.** The

Siren.—The instrument which is chiefly employed for counting the vibrations of a given sound is the *siren* of Cagniard de Latour. It is represented in Figs. 131, 132, the former being a front and the latter a back view.

There is a small wind chest, nearly cylindrical, having its top pierced with 15 holes, disposed at equal distances round the circumference of a circle. Just over this, and

Fig. 131.

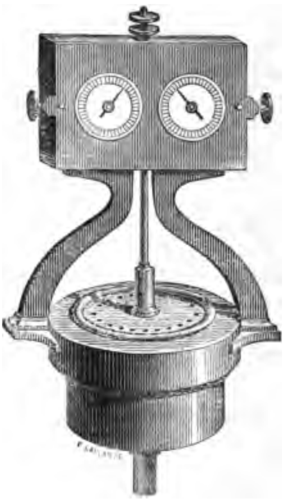
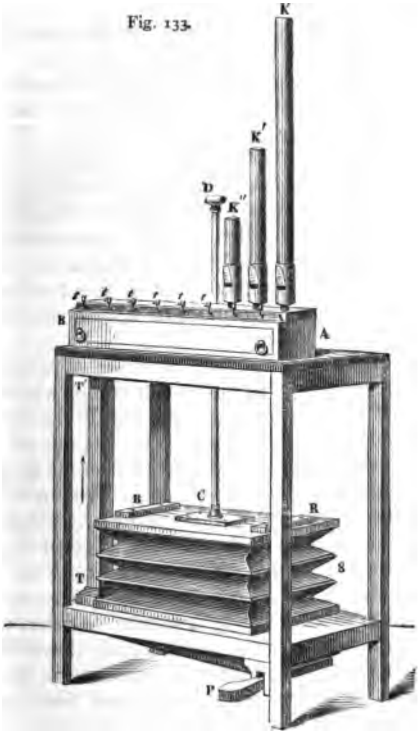


Fig. 132.

nearly touching it, is a movable circular plate, pierced with the same number of holes similarly arranged, and so mounted that it can rotate very freely about its centre, carrying with it the vertical axis to which it is attached. This rotation is effected by the action of the wind, which enters the wind-chest from below and escapes through the holes. The form of the holes is shown by the section in Fig. 132. They do not pass perpendicularly through

the plates, but slope contrary ways, so that the air, when forced through the holes in the lower plate, impinges ***upon one side*** of the holes in the upper plate, and thus blows it round in a definite direction. The instrument is driven

Fig. 133.



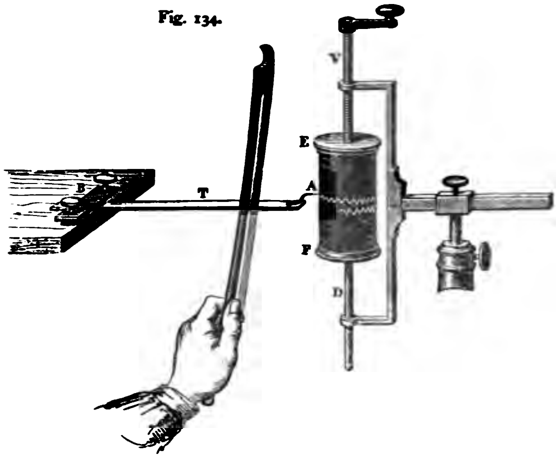
by means of the bellows shown in Fig. 133. As the rotation of one plate above the other causes the holes to be alternately opened and obstructed, the wind escapes in successive puffs, 15 to each revolution. A sound is thus

produced whose pitch is defined by the number of puffs in a second.—Fig. 132 shows the mechanism for registering the number of revolutions. The shaft of the revolving disk carries an endless screw, driving a wheel of 100 teeth, whose axle carries a hand traversing a dial marked with 100 divisions. Each revolution of the perforated disk causes this hand to advance one division. A second toothed wheel is driven intermittently by the first, advancing suddenly one tooth whenever the hand belonging to the first wheel passes the zero of its scale. This second wheel also carries a hand traversing a second dial, and at each of the sudden movements just described this hand advances one division. Each division accordingly indicates 100 revolutions of the disk, or 1500 vibrations. By pushing in one of the two buttons which are shown, one on each side of the box containing the toothed wheels, we can instantaneously connect or disconnect the endless screw and the first toothed wheel.

In order to determine the number of vibrations per second corresponding to any given sound which we have the power of maintaining steadily, we fix the siren on the bellows, leaving the recording apparatus out of gear, and drive the siren until the note which it emits is judged to be in unison with the given note. We then, either by regulating the pressure of the wind or by employing the finger to press with more or less friction against the revolving shaft, contrive to keep the note of the siren constantly in unison with the other for a measured interval of time, which we observe by a watch. At the commencement of the measured time we suddenly throw the recording apparatus into gear, and at its termination we suddenly throw it out of gear. The initial and final readings of the dials are noted down, units and tens being read from the one dial, and hundreds and thousands from the other, and their difference is the number of revolutions made in the measured interval of time. The number of vibrations is 15 times this, hence the number of vibrations per second can be computed.

399. **Vibroscope.**—The vibrations of a spring or a tuning-fork can be made to record themselves in the form of a sinuous curve by the method illustrated in Fig. 134. A light style A is attached to the end of the spring, and

Fig. 134.



a drum E F covered with smoked paper is fixed in such a position that the style just touches the surface of the paper. The spring is set in vibration by a fiddle-bow or otherwise, and the cylinder is then turned by means of its handle. The trace thus obtained is usually similar to the curve of art. 380, and each complete vibration of the spring gives one complete wave-form of the curve. The screw V on which the cylinder is mounted causes it to advance lengthwise when turned, and thus successive traces do not interfere.

This apparatus (called the vibroscope) is well adapted for comparing the numbers of vibrations made by two tuning-forks in the same time.

COMPOSITION OF VIBRATIONS.

400. Interference of Undulations.—When two sounds reach the same point in the air, the disturbance of density at this point is the sum of the disturbances due to the two sounds considered separately. For instance, if one sound produces at a given instant a compression amounting to ·003 of the undisturbed density, and the other a compression amounting to ·001, the actual compression will be ·004; or if one produces a compression of ·003, and the other an *extension* of ·001, the actual compression will be ·002.

In like manner, if the velocities of a given particle due to the two sounds at a given instant are respectively ·003 and ·001, the actual velocity of the particle will be ·004 if the velocities are in the same direction, and ·002 if they are in opposite directions. If they are neither in the same nor opposite directions, but make an angle with each other, the actual velocity will be found by the parallelogram of velocities.

401. Compression Associated with Forward Velocity.—Fig. 135 represents, in an exaggerated form, the successive phases of an undulation propagated through 7 particles A B C D E F *a* originally equidistant, the distance from the first to the last being one wave-length. The diagram is composed of 13 horizontal rows, the 13th being merely a repetition of the first, and the successive rows represent the positions of the particles at successive times, the interval of time from each row to the next being $\frac{1}{12}$ of the period of a vibration.

In the first row, A and *a* are points of maximum compression, and D is a point of maximum extension. In the third row, the maximum compression is at B and the maximum extension at E. In the fifth row the compression and extension have advanced by one more letter, and so on to the end of the diagram, which is precisely similar to the beginning.

The velocities of the particles can be estimated by the

comparison of successive rows. It is thus seen that the greatest forward velocity is at the points of maximum

Fig. 135.

```
    A B   C         D         E     F a
      AB  C       D         E      F  a
       ABC     D         E        F    a
        A  BC  D       E         F     a
        A    BCD    E        F         a
      A      B  CD  E       F          a
    A       B     CDE     F        a
  A        B       C DE   F       a
 A       B        C    DEF      a
A       B        C      D EF   a
 A    B        C        D    EFa
   A  B      C        D      E Fa
     AB    C        D        E   Fa
```

compression, that the greatest backward velocity is at the points of maximum extension, and that there is no velocity at the points of no compression or extension. Each particle has its greatest velocities, and greatest compression and extension, in passing through its mean position, and comes for an instant to rest in its extreme positions, which are also positions of mean density.

402. Interference of Waves Travelling in Opposite Directions.—This relation between density and velocity leads to results of special importance when two sets of equal and similar waves travel in opposite directions through the same air.

At the instants at which the two sets of waves concur as regards density, they are in complete opposition as regards velocity, because the forward direction for one set is opposite to the forward direction for the other. At these instants the compressions and extensions are double of those due to either system separately, and all the particles are at rest.

After the lapse of a quarter-period, the two sets of waves

will each have travelled a quarter wave-length, and their position relative to each other will have altered by a half wave-length. They will, therefore, be in complete opposition as regards density, and consequently in complete accordance as regards velocity. There will be neither extension nor compression anywhere, but the velocities at all points will be double of those due to either set of waves separately.

After the lapse of a second quarter-period, there will again be complete accordance as regards density, and opposition as regards velocity, but the maximum extensions and compressions will be half a wave-length distant from their positions at the instant first considered,—in other words, the extensions and compressions will have exchanged places.

After the lapse of a third quarter-period, there will again be uniform density, and velocities double of those due to either set of waves separately; but these velocities will be in the opposite directions to those which occurred at the same points half a period earlier.

After the lapse of a fourth quarter-period things will be in the same state as at first.

403. **Nodes and Antinodes.**—Undulation of this kind is called *stationary*. At certain fixed points called *nodes* there is permanent rest, with disturbance of density always double of that due to either set of waves separately. At other fixed points called *antinodes* there is permanent natural density, with velocity always double of that due to either set of waves separately. The distance between a node and the nearest antinode in either direction is a quarter wave-length.

The distance between two consecutive nodes, or between two consecutive antinodes, is half a wave-length. Two consecutive nodes are in constant opposition as regards density, and two consecutive antinodes are in constant opposition as regards velocity. In fact, the air moves towards a node from both sides simultaneously, thus producing compression at this node and extension

at the two neighbouring nodes, and then moves away on both sides simultaneously, thus producing the opposite effects.

404. Application to Pipes.—We now proceed to apply these principles to the vibrations of the air in pipes.

First. At the stopped end of a pipe there is a node; for at the instant of maximum density due to an incident wave, there is also maximum density due to the reflected wave.

Secondly. At the open end of a pipe there is an antinode; for at the instant of maximum density due to an incident wave, there is minimum density due to the reflected wave, inasmuch as the reflection is negative.

405. Tones of Open Pipe.—In a pipe open at both ends, there is an antinode at each end, and therefore the length of the pipe is either half a wave-length, or some multiple of half a wave-length.

In a pipe open at one end and stopped at the other, there is a node at one end and an antinode at the other, hence the length of the pipe is either a quarter of a wave-length, or an odd multiple of a quarter of a wave-length.

Let l denote in each case the length of the pipe, and λ the wave-length of one of its tones; then if the pipe be open at both ends, we have l equal either to $\frac{\lambda}{2}$, or $\frac{2\lambda}{2}$, or $\frac{3\lambda}{2}$, or $\frac{4\lambda}{2}$, or $\frac{5\lambda}{2}$, &c.; whence λ is equal either to $2l$, or $\frac{2l}{2}$, or $\frac{2l}{3}$, or $\frac{2l}{4}$, or $\frac{2l}{5}$, &c.; and if v denote the velocity of sound, the number of vibrations per second is either $\frac{v}{2l}$, or $\frac{2v}{2l}$, or $\frac{3v}{2l}$, or $\frac{4v}{2l}$, or $\frac{5v}{2l}$, &c., which are as the series of whole numbers 1, 2, 3, 4, 5, &c.

406. Tones of Stopped Pipe.—If the pipe be stopped at one end and open at the other, we have l equal either to $\frac{\lambda}{4}$, or $\frac{3\lambda}{4}$, or $\frac{5\lambda}{4}$, &c.; whence λ is equal either to $4l$ or $\frac{4l}{3}$, or $\frac{4l}{5}$, &c.; and the number of vibrations per second is either $\frac{v}{4l}$, or $\frac{3v}{4l}$, or $\frac{5v}{4l}$, &c., which are proportional to the series of odd numbers 1, 3, 5, &c.

407. Fundamental Tone and Overtones.—In each case the lowest tone is called the *fundamental tone*, and the others are called *overtones*. It it easy, by blowing with different degrees of strength, to produce several overtones from a pipe which is narrow in proportion to its length.

408. Practical Correction.—In wide pipes it is necessary to make allowance for lateral vibration, the effect of which is to make all the tones—and especially the overtones—deeper than they would be according to the above calculations.

409. Beats.—When two notes which are nearly, but not quite, in unison, are sounded together, a peculiar throbbing effect is produced; we hear bursts of sound, with intervals of comparative silence between them. The bursts of sound are called *beats*, and the notes are said to beat together. If we have the power of tuning one of the notes, we find that as they are brought more nearly into unison the beats become slower, and that, as the departure from unison is increased, the beats become more rapid, till they degenerate first into a rattle and then into a discord. The effect is most striking with deep notes.

These beats are explained by the interference of the two systems of sound waves. Let one source of sound, for example, send to the ear 100 waves per second, and the other 101. Then, starting from an instant when the changes of density in the air close to the ear of the listener are concurrent, one of the two systems of waves gradually falls behind the other at the rate of one wave-length per second. After the lapse of half a second they will be in complete opposition, and after another half-second they will again be in complete accordance. A beat is heard when they are in accordance, and extinction occurs when they are in opposition. The number of beats is therefore equal to the number of vibrations that one source gains upon the other.

410. Theoretical Computation of Velocity of Sound.—The velocity of sound in fluids (whether liquids

or gases) depends on the two elements *density* and *resistance to compression*. It is inversely proportional to the square root of the density, and directly proportional to the square root of the resistance which the fluid opposes to being compressed.

If a pressure p applied to the fluid produces a compression v (that is, reduces unit volume to $1 - v$), then $\frac{p}{v}$ is called the *resistance to compression*, or the *coefficient of elasticity*. Let this be denoted by E; then the velocity of sound in the fluid (when proper units are employed) will be $\sqrt{\frac{E}{D}}$, D denoting its density.

411 **Application to Air.**—When air is compressed *at constant temperature*, its resistance to compression can be calculated by Boyle's law; but when it is compressed *adiabatically*,—that is, compressed without any interchange of heat with surrounding bodies, its temperature rises, and its resistance to compression is 1·41 times the resistance at constant temperature.

The compressions and extensions which accompany the propagation of sound are so transient that there is no time for the small differences of temperature which they produce to be diminished by conduction. Hence the adiabatic resistance to compression is that which must be employed in calculating the velocity of sound, and the expression for this velocity (when proper units are employed) is $\sqrt{(1{\cdot}41\,\frac{P}{D})}$, P denoting the pressure of the undisturbed air, and D its density.

Variations of barometric pressure do not affect $\frac{P}{D}$, since pressure varies as density when temperature is constant; hence they do not affect the velocity of sound; but high temperature makes P large or D small, and thus sound travels faster at high than at low temperatures. The velocity in fact is proportional to the square root of $1 + at$ (art. 182).

412. Velocity in Gases determined by Experiments with Organ-pipes.—The pitch of an organ-pipe (except reed-pipes) rises with the temperature of the air; for the number of vibrations n made in unit time is equal to $\frac{v}{\lambda}$, v denoting velocity and λ wave-length. But λ is sensibly the same at all temperatures, hence n is simply proportional to v.

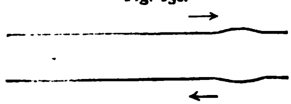
On the same principle, the velocity of sound in any gas can be compared with its velocity in air; for if a pipe gives a note of n vibrations in air and of n' vibrations in the gas, the velocity of sound in air is to that in the gas as n to n'. In this way, the velocities in the principal gases have been determined, and have been found to be nearly in the inverse ratio of the square roots of their densities when compared at equal pressures.

The departure from exactness is sufficiently decided to show that the ratio of the two coefficients of elasticity, which is about 1·41 for air, has slightly different values for some other gases.

VIBRATIONS OF STRINGS.

413. Velocity of Pulses.—The vibration of a musical string, like that of a column of air, can be resolved into two coexistent series of equal waves, steadily travelling along the string in opposite directions, and reflected, with reversal of form, from the ends, as in Fig. 136.

Fig. 136.



A pulse started from any part of the string will have travelled over twice the length of the string before returning to its original position and form.

The velocity of a pulse along a musical string can be calculated (when proper units are employed) by the formula

$$v = \sqrt{\frac{t}{m}}; \qquad \text{(A)}$$

where t denotes the tension of the string, m the mass of the string per unit of length, and v the velocity of the pulse.

414. Pitch of Fundamental Tone.—Let n denote the vibration number, that is, the number of vibrations made per second, λ the wave-length of either of the two series of waves which are travelling along the string in opposite directions, and l the length of the string; then by the general formula (art. 379) we have

$$n = \frac{v}{\lambda}. \qquad \text{(B)}$$

Putting for v its value $\sqrt{\frac{t}{m}}$ and for λ its value (for the fundamental tone) $2\,l$, we have

$$n = \frac{1}{2l}\sqrt{\frac{t}{m}}. \qquad \text{(C)}$$

This formula involves the following laws:—

1. When the length l of the vibrating portion of the string (for this is the strict meaning of l) is altered without changing the tension, the vibration-number varies inversely as l.

2. When the tension is altered without sensible change of the length or mass of the vibrating portion, the vibration-number varies as the square root of the tension.

3. If the mass of a string be altered (for example by winding fine wire round it) without altering its tension, the vibration-number varies inversely as the square root of the mass.

4. Strings of the same material and of the same length will give the same note if they are stretched with forces directly proportional to their sectional areas.

The first law is illustrated by the fingering of the violin, the pitch being raised as the portion of string between the finger and the bridge is shortened.

The second is illustrated by the mode of tuning, which consists in tightening the string when its pitch needs to be raised.

The third is illustrated by the construction of the bass string.

The fourth is illustrated by the sizes and tensions of the other three strings. The treble string, though the smallest, has to bear the strongest pull.

415. Overtones of Strings.—Besides the fundamental mode of vibration (which yields the fundamental tone), a string has several other modes of *free* vibration (that is, of vibration which can be sustained for some time by the elasticity of the string when once started). In these modes of vibration there are nodes (that is, places of rest) not only at the two ends, but at one or more intermediate points which divide the string into equal portions.

When a string vibrates in its fundamental mode pure and simple, its extreme positions are as represented by the continuous and the dotted line in Fig. 137. When it

Fig. 137.

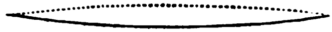


Fig. 138.

vibrates in such a way as to give its second tone (or first overtone) its extreme positions are as represented by the continuous line ACNDB and the dotted line AC'ND'B in Fig. 138. The point N is held at rest by the equal and opposite pulls at its two sides, and each of the two halves of the string vibrates on its own account as if it were attached to a fixed point of support at N. The tone emitted is accordingly that due to a string of half the length of A B, and has twice as many vibrations per second as the fundamental tone.

This mode of vibration can be induced by bowing either half of the string with a fiddle-bow, while the point N is lightly touched with the finger; and when it has

thus been induced it will be kept up for some time after both the bow and the finger are removed.

In like manner by lightly touching it at $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., of its length, the string can be made to divide into 3, 4, 5, &c., vibrating segments, and emit notes whose vibration-numbers are 3, 4, 5, &c., times that of the fundamental. The bow should be applied to one of the two extreme segments.

In every case, the time of a vibration is the time of swinging from one extreme position to the other *and back again*, and the motion may either be in one plane or round and round, spindle-fashion.

416. Compound Vibrations.—The vibrations of strings as actually employed in music, are usually compounded of several of these modes of free vibration. The middle and lower notes of a piano contain four or five overtones blended with the fundamental, and the notes of a violin contain a still larger number.

417. Sounding Boards.—Strings have so small a surface of contact with the air that, in order to produce sufficient sound, they require to be connected with some body of large surface capable of readily taking up their vibrations and transmitting them to the air. Wood, either in the form of a thin board (as the sounding-board of a piano,) or of a light box (as the body of a violin), is usually employed for this purpose. The action is somewhat analogous to the resonance of a column of air to a tuning-fork, but with the great difference that the column of air only resounds to certain special notes, whereas the board or box resounds to all notes lying within a wide compass, though rather more readily to some than to others.

VIBRATIONS OF RODS AND PLATES.

418. Vibrations of Rods.—A wooden rod, when held in the middle, and rubbed about half way between the middle and one end with chamois leather covered with powdered rosin, emits a musical note due to longi-

tudinal vibration of the rod. The rod, in fact, alternately stretches and contracts, the middle remaining stationary, while the two ends recede from it together and approach it together. The cross section at the middle is therefore a node, and the ends are antinodes.

419. Determination of Velocity of Sound in Solids.—The time of a complete vibration is, just as in the case of a string or an open organ-pipe, the time that a pulse would occupy in travelling over twice the length of the rod. Hence, from observing the pitch of the note emitted, the velocity with which longitudinal pulses travel along the rod can be inferred. For example, if the note be C of 512 vibrations per second, and the length of the rod be 10 feet, a length of 20 feet is travelled over 512 times in a second, and the velocity is 10,240 feet per second. This is one of the most convenient practical methods of determining the velocity of sound in solid bodies.

The overtones of a rod follow the same law as those of an open organ-pipe.

420. Vibrations of Plates.—The existence of nodes in a vibrating body is beautifully shown by firmly fixing a square plate of metal in the middle, and bowing its edge with a well rosined double-bass or violoncello bow. By varying the bowing, the plate can be made to give several distinct notes; and if sand be sifted over the plate, it quickly settles on the nodal lines in each case. For the deepest tone, the nodal lines divide the plate into four equal squares; for the next tone they divide it into four equal triangles; and for some of the higher tones they form very elaborate figures. To obtain any particular figure known to be among those which the plate is capable of giving, the finger should be applied to one of the nodal lines of this figure, and the bow should be applied about midway between two nodal lines.

QUALITY OF MUSICAL SOUNDS.

421. Sounds of the same pitch and loudness are often very unlike; in other words, they may differ widely in *quality*.

422. **Harmonics.**—In order to explain the physical basis of difference in quality, we must first define the *harmonics* of a note. They are a series of notes whose vibration-numbers are respectively double, triple, quadruple, &c., of the vibration-number of the given note.

A musical note consists, in the great majority of cases, of a fundamental tone blended with some of its harmonics; and it is upon the relative strengths of these harmonics, as compared with the fundamental tone and with each other, that the quality of the note depends.

A rich and mellow note usually contains the first four or five harmonics, gradually diminishing in strength as they are further from the fundamental, the fundamental itself being stronger than any. A note of penetrating metallic ring is strong in the higher harmonics; and if it grates upon the ear, this is often owing to the circumstance that very high harmonics are discordant with one another. Bodies whose overtones are not harmonics, such as bells and plates, usually emit sounds in which these overtones are blended and produce discordance.

423. **Origin of Harmonics.**—The origin of harmonics is to be found sometimes in the different modes of free vibration of the body which emits the sound. This is especially the case with the sounds of stringed instruments. But in other cases (including the tones of the siren and the human voice) it depends on a very different cause, namely, on the mathematical law that every series of precisely similar vibrations is either simple (that is, similar to the small vibrations of a pendulum) or compounded of one set of simple vibrations giving the fundamental tone and other sets of simple vibrations giving the harmonics. Every compound tone consists of simple tones, each of which is due to simple vibrations.

424. **Loudness.**—The loudness or intensity of a sound

is measured *physically* by the amount of energy which it conveys to the ear in a given time; and this energy, in comparing two simple sounds, is proportional to the square of the amplitudes of the particles of air in the neighbourhood of the ear.

But from the point of view of *sensation*, this rule of comparison can only be admitted when the two simple sounds compared are of the same pitch; for the ear is unequally sensitive to simple sounds of different pitches. Sounds may be so high in pitch as not to be heard at all by the human ear; and within the limits of audibility there is considerable difference in sensibility. Within the range of pitch employed in music, the ear is more sensitive to sounds of high than of low pitch; that is to say, the same amount of energy of aerial vibration produces a more intense sensation when the pitch is high than when it is low. Of two compound tones of equal energy, that which is strongest in high harmonics will generally affect the ear the most.

425. Law of Inverse Squares.—When sound is emitted from a point in mid air and meets with no obstruction on its way, the amplitude of its vibrations diminishes inversely as the distance they have been propagated, and its loudness diminishes inversely as the square of the distance. In fact, the same amount of energy which the source emits in unit time, must cross in unit time the circumference of every sphere described round it as centre; these surfaces are as the squares of the distances, and therefore the energy per unit area must be inversely as the square of the distance.

426. Extinction.—This, however, is on the assumption that onward propagation involves no loss of sonorous energy,—an assumption which is not strictly true, inasmuch as the vibration of the air implies friction, and friction implies generation of heat at the expense of the energy of vibration. Sonorous energy must therefore diminish with distance rather faster than according to the law of inverse squares.

All sound in becoming extinct becomes converted into heat. This conversion is greatly promoted by defect of homogeneity in the medium of propagation. In a fog or a snow-storm, the liquid or solid particles present in the air produce innumerable reflections, in each of which a little sonorous energy is converted into heat.

427. **Propagation through a Tube.**—When sound is propagated through a uniform tube, the law of inverse squares does not apply. The intensity in this case would be the same at all distances, were it not in the first place for friction, and in the second place for the transmission of a little sonorous energy through the sides of the tube.

COMPARISON BETWEEN SOUND AND LIGHT.

428. Both sound and light consist in vibratory movement which is propagated from the source through an elastic medium. The waves of light are incomparably shorter than those of sound, and the velocity of their propagation is incomparably greater. Sound-waves of different lengths travel with the same velocity. Luminous waves of different lengths travel with the same velocity *in vacuo*, but in liquids and solids (and to an almost inappreciable extent in gases) those of shortest wave-length travel slowest.

The refraction of a ray of light in passing out of one medium into another, depends upon the difference of its velocities in the two media. It travels fastest *in vacuo*, and slowest in those media which are most highly refractive, the index of refraction being equal to the ratio of the velocities in the two media.

The extreme red rays have nearly double the wave-length of the extreme violet.

The separation of rays of different wave-lengths by a glass prism is a direct consequence of the fact that the shortest waves are the most retarded by the action of the glass.

429. The lengths of luminous waves can be measured

with considerable accuracy by observations on "diffraction" —a subject which does not come within the scope of the present treatise; and by dividing the wave-length into the distance travelled over in a second, we obtain the number of vibrations per second. Red light makes about 400 millions of millions of vibrations per second, and violet light nearly twice as many. In passing out of one medium into another the rapidity of vibration remains unaltered, but the wave-length changes in the same ratio as the velocity. This remark is true of sound as well as light.

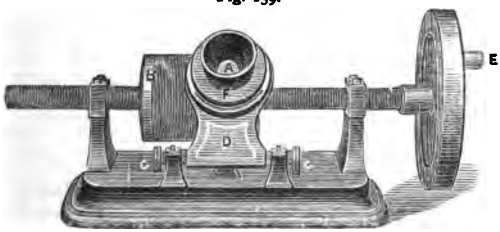
430. Sound cannot be propagated through a vacuum. Light, on the other hand, is propagated for immense distances through the interplanetary and interstellar spaces, which are more perfect vacua than any that can be artificially obtained. Hence it is inferred that light is propagated by the vibrations of some medium different from ordinary matter. The immense velocity of light shows that this medium is incomparably more resilient in proportion to its density than any of the forms of ordinary matter. This medium, whose existence is now universally accepted by physicists, is called the *luminiferous ether*.

431. There is another difference between sound and light in the direction of their vibrations. The vibrations by which sound is propagated through gases and liquids, and probably also through solids, are longitudinal,—that is to say, each particle of the medium travels backwards and forwards along a line which points in the direction in which the sound is travelling; there is no transverse motion. In luminous vibration, on the contrary, the most important part of the motion is transverse, as is proved by the phenomena of polarized light.

432. **Phonograph.**—Mr. Edison of New York has been successful in constructing an instrument which can reproduce articulate sounds spoken into it. The voice of the speaker is directed into a funnel, which converges the sonorous waves upon a diaphragm carrying a style.

The vibrations of the diaphragm are impressed by means of this style upon a sheet of tin-foil, which is fixed on the outside of a cylinder to which a spiral motion is given as in the vibroscope. After this has been done, the cylinder with the tin-foil on it is shifted back to its original position, the style is brought into contact with the tin-foil as at first, and the cylinder is then turned as before. The indented record is thus passed beneath the style, and forces it and the attached diaphragm to execute movements resembling their original movements. The diaphragm accordingly emits sounds which are imitations of those previously spoken to it. Tunes sung into the funnel are thus reproduced with great fidelity, and sentences clearly spoken into it are reproduced with sufficient distinctness to be understood.

Fig. 139.



The instrument is represented in Fig. 139. By turning the handle E, which is attached to a massive fly-wheel, the cylinder B is made to revolve and at the same time to travel longitudinally, as the axle on which it is mounted is a screw working in a fixed nut. The surface of the cylinder is also fluted screw-fashion, the distance between its flutings being the same as the distance between the threads on the axle. A is the diaphragm, of thin sheet-iron, having the style fixed to its centre but not visible in the figure. The diaphragm and funnel are carried by the frame D, which turns on a hinge at the bottom. C C are adjusting screws for bringing the style

exactly opposite the centre of the groove on the cylinder, and another screw is provided beneath the frame D, for making the style project so far as to indent the tin-foil without piercing it. The tin-foil is put round the cylinder and lightly fastened with cement, so that it can be quickly taken off and changed.

In another form of the instrument the rotation of the cylinder is effected by means of a driving weight and governor, which give it a constant velocity. This is a great advantage in reproducing music, but is of little or no benefit for speech.

EXAMPLES IN SOUND.

XXXIV.—Period, Wave-length, and Velocity. Arts. 378–379.

1. If an undulation travels at the rate of 100 feet per second, and the wave-length is 2 ft., find the period of vibration of a particle and the number of vibrations which a particle makes per second.

2. It is observed that waves pass a given point once in every 5 seconds, and that the distance from crest to crest is 20 ft. Find the velocity of the waves in ft. per sec.

3. The lowest and highest notes of the normal human voice have about 80 and 800 vibrations respectively per second. Find their wave-lengths when the velocity of sound is 1100 ft. per sec.

4. Find their wave-lengths in water in which the velocity of sound is 4900 feet per sec.

5. Find the wave-length of a note of 500 vibrations per sec. in steel in which the velocity of propagation is 15,000 ft. per sec.

XXXV.—Musical Intervals. Arts. 392–396.

1. Show that a "fifth" added to a "fourth" makes an octave.

2. Calling the successive notes of the gamut (as defined in art. 396) Do_1, Re_1, Mi_1, Fa_1, Sol_1, La_1, Si_1, Do_2, show that the interval from Sol_1 to Re_2 is a true "fifth."

3. Find the first 5 harmonics of Do_1 (see art. 422).

XXXVI.—Siren. Art. 398.

1. A siren of 15 holes makes 2188 revolutions in a minute when in unison with a certain tuning-fork. Find the number of vibrations per second made by the fork.

2. A siren of 15 holes makes 440 revolutions in a quarter of a minute when in unison with a certain pipe. Find the note of the pipe (in vibrations per second).

XXXVII.—Reflection of Sound, and Tones of Pipes. Arts. 383–389, 402–408.

1. Find the distance of an obstacle which sends back the echo of a sound to the source in $1\frac{1}{2}$ seconds, when the velocity of sound is 1100 ft. per sec.

2. A well is 210 ft. deep to the surface of the water. What time will elapse between producing a sound at its mouth and hearing the echo?

3. What is the wave-length of the fundamental note of an open organ-pipe 16 ft. long; and what are the wave-lengths of its first two overtones? Find also their vibration-numbers per second.

4. What is the wave-length of the fundamental tone of a stopped organ-pipe 5 ft. long; and what are the wave-lengths of its first two overtones? Find also their vibration-numbers per second.

5. What should be the length of a tube stopped at one end that it may resound to the note of a tuning-fork which makes 520 vibrations per second; and what should be the length of a tube open at both ends that it may resound to the same fork. [The tubes are supposed narrow, and the smallest length that will suffice is intended.]

6. Would tubes twice as long as those found in last question resound to the fork? Would tubes three times as long?

XXXVIII.—Beats. Art. 409.

1. One fork makes 256 vibrations per second, and

another makes 260. How many beats will they give in a second when sounding together?

2. Two sounds, each consisting of a fundamental tone with its first two harmonics (art. 422), reach the ear together. One of the fundamental tones has 300 and the other 302 vibrations per second How many beats per second are due to the fundamental tones, how many to the first harmonics, and how many to the second harmonics?

3. A note of 225 vibrations per second, and another of 336 vibrations per second, are sounded together. Each of the two notes contains the first two harmonics of the fundamental. Show that two of the harmonics will yield beats at the rate of 3 per second.

XXXIX.—Velocity of Sound in Gases. Arts. 410–412.

1 If the velocity of sound in air at 0° C. is 33,240 cm. per sec., find its velocity in air at 10° C., and in air at 100° C.

2. If the velocity of sound in air at 0° C. is 1090 ft. per sec., what is the velocity in air at 10°?

3. Show that the difference of velocity for 1° of difference of temperature in the Fahrenheit scale is about 1 ft. per sec.

4. If the wave-length of a certain note be 1 metre in air at 0°, what is it in air at 10°?

5. The density of hydrogen being ·06926 of that of air at the same pressure and temperature, find the velocity of sound in hydrogen at a temperature at which the velocity in air is 1100 ft. per sec.

6. If a pipe gives a note of 512 vibrations per second in air, what note will it give in hydrogen?

XL.—Vibrations of Strings. Arts. 413–417.

1. A string makes 400 vibrations per sec. when stretched with a force of 20 lbs. How many will it make when

stretched with 45 lbs., and what will be the interval between the two notes produced? [The increase of length with the greater weight is supposed negligible.]

2. A steel wire and a catgut string have the same diameter and are stretched with equal forces. Supposing steel to be 9 times as dense as catgut, find the ratio of their numbers of vibrations, and the interval between their notes. What should be the ratio of the stretching weights to make them give the same note?

3. If the length of a string and the stretching weight be each increased fourfold, what will be the effect on the pitch?

XLI.—Longitudinal Vibrations of Rods. Arts. 418–419.

1. If sound travels along fir in the direction ot the fibres at the rate of 15,000 ft. per sec., what must be the length of a fir rod that, when vibrating longitudinally in its fundamental mode, it may emit a note of 750 vibrations per second.

2. A rod 8 ft. long, vibrating longitudinally in its fundamental mode, gives a note of 800 vibrations per second. Find the velocity with which pulses are propagated along it.

ANSWERS.

XXXIV.—Ex. 1. $\frac{1}{50}$ sec., 50. Ex. 2. 4. Ex. 3. $13\frac{3}{4}$ ft., $1\frac{3}{8}$ ft. Ex. 4. $61\frac{1}{4}$ ft., $6\frac{1}{8}$ ft. Ex. 5. 30 ft.

XXXV.—Ex. 1. $\frac{3}{2} \times \frac{4}{3} = 2$. Ex. 2. $\frac{256}{384} = \frac{2}{3}$. Ex. 3. Do_2, Sol_2, Do_3, Mi_3, Sol_3.

XXXVI.—Ex 1. 547. Ex. 2. 440.

XXXVII.—Ex. 1. 825. Ex. 2. $\frac{21}{55} = \cdot382$ sec. Ex. 3. 32 ft., 16 ft., $10\frac{2}{3}$ ft.; $34\frac{3}{8}$, $68\frac{3}{4}$, $103\frac{1}{8}$. Ex. 4. 20 ft., $6\frac{2}{3}$ ft., 4 ft.; 55, 165, 275. Ex. 5. $\frac{55}{104}$ ft., $\frac{55}{52}$ ft. Ex. 6. An open tube twice or three times as long will resound, because one of its over-

tones will coincide with the note of the fork. A stopped tube three times as long will resound, but a stopped tube twice as long will not.

XXXVIII.—Ex. 1. 4. Ex. 2. 2, 4, 6. Ex. 3. $675 - 672 = 3$.

XXXIX.—Ex. 1. 33843, 38850. Ex. 2. 1110. Ex. 3. The velocity is 1090 at 32° and 1110 at 50°. Ex. 4. 1·018 metre. Ex. 5. 4180 ft. per sec. Ex. 6. 1945 vibrations per sec.

XL.—Ex. 1. 600 vibrations. A fifth. Ex. 2. As 1 to 3, the catgut being the quickest; octave and fifth; as 9 to 1. Ex. 3. Lowered an octave.

XLI.—Ex. 1. 10 ft. Ex. 2. 12800 ft. per sec.

ELECTRICITY.

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433. Excitement by Friction.—A piece of rosin or sealing wax, after being rubbed with dry flannel or woollen cloth, is found to have the property of attracting small pieces of paper or other light bodies.

If a pith ball is suspended by a thread of dry unbleached silk, it is found that the rosin or wax, after being rubbed, attracts the pith ball until contact ensues, and then repels it.

A glass rod, after being rubbed with a piece of silk (care being taken that both glass and silk are very dry), also attracts the pith ball, and after contact repels it.

If the operator holds the rosin in one hand and the glass in the other, he will find that when one of them repels the ball the other attracts it.

434. The rosin and the glass in these experiments are said to be electrified. An electrified body exhibits special properties of attraction and repulsion, and every electrified body belongs to one or the other of two classes, represented respectively by the glass and the rosin in the above experiments. To decide to which class it belongs, we have only to compare its action with one of these two bodies—say with rosin rubbed with flannel. If the pith ball is simultaneously repelled by the rosin and the body in question, the electrical condition of the body resembles that of the rosin, and the body is said to be *resinously* or *negatively* electrified. If, on the other hand, the body repels the pith ball at the same time that the rosin attracts it, and attracts the ball when the rosin repels it, the electrical condition of the body is similar to that of the glass rod in the above experiment, and the body is said to be *vitreously* or *positively* electrified.

An electrified body is said to have electricity residing upon it, or to be charged with electricity. Electricity, therefore (whatever its real nature may be), is of two opposite kinds, *vitreous* or *positive*, which resides upon smooth glass rubbed with silk, and *resinous* or *negative*, which resides upon rosin rubbed with flannel. We are not entitled to assert that electricity is a fluid or two fluids; nor that it is a substance, or two substances, any more than we are entitled to assert that light and sound are substances.

435. The flannel with which the rosin has been rubbed will be found to be vitreously electrified; and the silk with which the glass has been rubbed will be found to be resinously electrified. Whenever electricity is excited by rubbing two bodies together, the electrifications produced in the two bodies are opposite.

Bodies can be roughly arranged in an order such that, speaking broadly, if any two of them be rubbed together, the one which stands earlier in the list will become positively and the other negatively electrified. The following is such a list:—

Fur of cat.	Paper.
Polished glass.	Silk.
Woollen stuffs.	Shellac.
Feathers.	Rough glass.
Wood.	

436. The terms "positive" and "negative" are the remains of a theory broached by Franklin, according to which a positively electrified body possesses more, and a negatively electrified body less than its natural share of one universally diffused electric fluid. The terms are eminently convenient on account of the direct opposition of properties, which is exactly expressed by difference of algebraic sign. It is, however, not to be imagined that either of the two kinds of electricity has more real and positive existence than the other. The attaching of the name positive to the kind which is otherwise called vitreous rather than to that which is called resinous is a matter of accident.

437. Conductors and Non-conductors.—Metals readily allow electricity to escape through them. Glass, rosin, silk, wool, indian-rubber, and gutta-percha oppose great resistance to its passage. The former are therefore called *conductors* and the latter *non-conductors* of electricity. The resistance of a non-conductor is many millions of millions of times greater than that of a metal.

A body is said to be insulated when it is not in contact with any other bodies except non-conductors; for example, a pith ball suspended by a silk thread, or a metallic ball supported on a glass stem, is insulated.

438. Like Electricities Repel, Unlike Attract.—Positive electricity repels positive and attracts negative electricity. In like manner, negative electricity repels negative and attracts positive.

A *quantity of electricity* is measured by the repulsion and attraction which it is capable of exerting. The unit quantity is that which, if collected at a point, would exert unit force upon an equal quantity at unit distance. The force varies inversely as the square of the distance, so that at a mutual distance of 2 units the force exerted by one unit quantity upon another would be $\frac{1}{4}$ of the unit force.

INDUCTION.

439. Induction.—The attractions and repulsions of electricity show themselves in two ways; they produce movements of electrified bodies, and they also produce movement of electricity itself in or upon electrified bodies. Thus, if two conducting balls electrified, one positively and the other negatively, are held near each other, not only will the balls attract each other, but a portion of the charge of each will leave the far side and come over to the near side in obedience to the attraction of the dissimilar electricity of the other ball. This kind of action is called *induction*, and sometimes, for distinctness, *electrostatic induction*.

440. When a positively electrified body is held near an insulated and uncharged pith ball, the latter becomes

charged with negative electricity on the near side, and positive on the far side,—the quantities of the two kinds being exactly equal.

An uncharged body may be regarded as containing an unlimited store of both electricities, exactly equal in amount. When equal quantities of positive and negative are artificially given to a body, the body is found to have no charge. A gain of one kind of electricity is not distinguishable from an equal loss of the other kind.

441. The attraction which is observed between electrified and (apparently) unelectrified bodies is due to the inductive action here described. The electrified body induces electricity unlike its own on the near side, and (if the body be insulated) electricity like its own on the far side. As force diminishes with increase of distance, the attraction upon the electricity on the near side prevails over the repulsion of that on the remote side. If the body is not insulated, the attraction will be still more powerful, because some or all of the electricity of like sign is repelled out of the body into the conductors behind it, so that the body as a whole has a charge of unlike sign to that of the influencing body.

442. **Gold-leaf Electroscope.**—An instrument for indicating the presence of electricity is called an *electroscope*. The gold-leaf electroscope has two gold leaves hung from the lower end of a metallic rod, whose upper end terminates in a knob or plate, also of metal. The leaves are protected by a glass shade.

The gold leaves, rod, and knob or plate may be regarded as one conductor. When a positively electrified body is held over the top, induction takes place in the conductor, negative electricity being attracted to the top, and positive repelled to the gold leaves at the bottom. The gold leaves being thus charged with like electricities repel each other.

443. If the influencing body is now removed, the leaves come together again (unless there has been leakage), their induced charge ceasing with the cause which produced it.

But if, while the influencing body is in position, the knob is for a moment touched with the finger, and the influencing body is then removed, the leaves will have acquired a permanent charge opposite in sign to that of the influencing body. This is an example of the principle mentioned above, that an uninsulated body acquires by induction a total charge opposite in sign to that of the influencing body.

It is often thought paradoxical that touching the knob, which is at the time charged with negative electricity by induction (if the influencing body be positive), should give it a further negative charge; but such is the fact, and it can be explained by the consideration that the finger as well as the knob is under the influence of the inducing body, and that the finger being uninsulated is more free to become negatively electrified by induction than the knob, which is part of an insulated conductor.

As long as the contact of the finger is kept up, the leaves hang straight down, and they remain in this position after the finger is taken away provided the influencing body is not moved. Movement of the influencing body either way makes the leaves separate. If it is moved nearer, the leaves become charged with electricity similar to that of the influencing body. If it is moved further away they become charged with the opposite kind.

444. In the actual use of the electroscope, it is usual to begin by thus giving it a charge by induction. If an electrified body held over it produces increased separation between the leaves, the inference is that the charge of this body is similar to that which has been given to the electroscope.

Caution, however, must be employed, or a false inference may be drawn. If a body electrified dissimilarly to the electroscope is held over it, the leaves at first approach each other, and they continue to approach till the body comes to a certain distance, when they hang down parallel. If the body is brought still nearer, they again repel, but with opposite electricity to that with which they

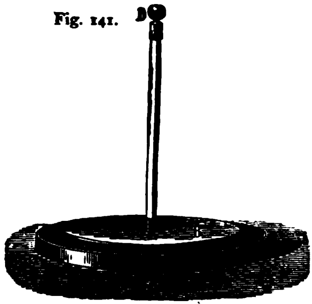
repelled before. In such cases, the first movement of the leaves (that is, their approach) is that which gives the indication of sign for the charge of the influencing body.

But again, if the hand, or any other conductor connected with the earth, is held over the top of the electroscope, the charge of the instrument induces an opposite charge in the body so held, and the reaction of this charge produces a diminution in the repulsion of the leaves. Diminished repulsion is therefore, to a certain extent, an uncertain test; there is danger of the observer attributing to an insulated body held in his hand an effect which may be really due to this influence of his hand.

445. Fig. 140 represents one form of the gold-leaf

Fig. 140.

Fig. 141.



electroscope. The leaves $a\,b$ are represented as divergent, and therefore as charged. The metallic columns cd are connected with the ground, and, becoming charged by induction with electricity opposite to that of the leaves, tend to increase their divergence. The small vessel between them contains chloride of calcium for maintaining a dry atmosphere in the interior.

446. **Electrophorus.**—The electrophorus (Fig. 141) is a convenient instrument for obtaining moderate charges of electricity. It consists essentially of a disk of ebonite,

shellac, or some other substance easily electrified by friction, and of a metallic plate which can be laid upon it and lifted off by an insulating handle. There is also on the under side of the excitable plate, and attached to it, another metallic plate or sheet called the sole-plate.

To set the instrument in action, friction must be employed to electrify the non-conducting disk,—whipping it with a catskin or with a piece of flannel is the usual method. Then the conducting plate must be laid upon it, and touched with the finger in order to connect it with the earth, or (what amounts to the same thing) with the sole-plate. In some forms of the instrument, the mere laying of the conducting upon the non-conducting plate makes contact with the sole-plate, and the touching with the finger is then unnecessary. The conducting plate is then lifted off by its insulating handle, and is found to be so strongly charged that it will give a spark when presented to a conductor. It can then be charged again by applying it a second time in like manner to the non-conducting plate (without repeating the friction of the latter), and will give a second spark not sensibly weaker than the first. If the apparatus is good and the weather favourable, the process may be repeated almost any number of times without the necessity of again exciting the non-conducting plate.

447. The explanation of the action is that the conducting plate, as often as it is applied to the excited plate and connected with the earth, acquires by induction a charge opposite in sign to that of the excited plate. Why induction takes place rather than communication of charge between the two plates in contact is not easily explained. Such action would not occur if both the plates were conductors, but non-conductors do not readily part with their electricity.

DISTRIBUTION OF ELECTRICITY.

448. The charge of a conductor is not diffused through its substance, but resides solely on its surface; and if

the conductor is hollow, so as to have two surfaces, one external and the other internal, the charge resides solely on the external surface. Nor is it, generally speaking, uniformly distributed even over the external surface; it is most abundantly collected at those parts which are most convex, or which project furthest beyond the general outline. If the conductor has any sharp points or edges, the electricity resides upon them in such abundance that it is very apt to break away through the air and leave the body, in virtue of the intensity of its own self-repulsion.

The laws of electric distribution are not independent laws, but are mathematical consequences of a repulsion varying inversely as the square of the distance.

Conductors which are intended to hold their charges without leakage, should have smooth surfaces, and should have all corners and edges rounded off. On the other hand, a conductor should be furnished with pointed projections, if we wish it to give off or receive electricity readily through the air. It must be borne in mind that to give off electricity of one sign is the same thing as to receive electricity of the opposite sign. A pointed conductor with no charge of its own is capable of drawing off electricity from any charged conductor or non-conductor to which it may be presented.

449. The *electric density* on any small element of the surface of a charged conductor signifies the quantity of electricity per unit area at this point. The quantity of electricity on a small area containing the point is approximately equal to the product of the area by the density at the point.

ELECTRICAL MACHINE.

450. In the ordinary frictional electrical machine, there is a glass plate (or sometimes a glass cylinder) which is made to revolve while tightly pressed against a cushion covered with an amalgam of tin and zinc. The friction of the glass against this rubber generates electricity, posi-

tive going to the glass and negative to the cushion. After leaving the cushion, the electrified surface of the glass passes a row of brass points attached to the "prime conductor," which is a large piece of hollow brass of rounded form, supported on a glass stem. These points draw off electricity from the glass plate to the prime conductor, which soon becomes highly charged.

It is necessary to provide for the escape of the negative electricity from the rubber, which would otherwise become so strongly negative that it would cease to fulfil its function. The rubber is therefore, in the ordinary use of the machine, connected with the earth by a brass chain, through which a continual stream of negative electricity flows off from the cushion to the earth, or, what is the same thing, a continual stream of positive electricity flows through it from the earth to the cushion.

In some machines, there is a second brass conductor supported on a glass stem, and connected with the rubber. Negative electricity can be drawn off from this conductor, if an exit is afforded to the positive electricity which goes to the positive conductor. When it is simply desired to obtain a negative charge, the positive conductor may be connected with the earth by a chain. When it is desired to obtain both kinds of electricity, they can be drawn simultaneously one from each conductor.

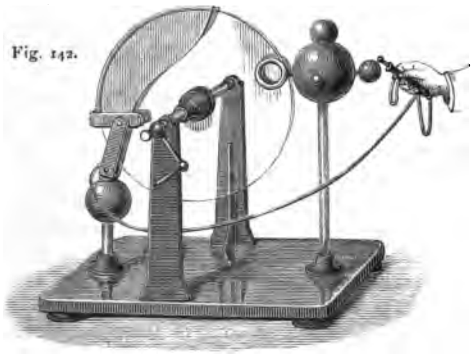
When an electrical machine is in good working order, strong sparks can be drawn from its conductor or conductors, their length varying from half an inch to several inches, according to the power of the machine and the condition of the air.

To obtain efficient working, it is usually necessary in this climate to place the machine near a fire. Glass readily contracts a deposit of moisture from the air, which spoils the insulation of the supports, besides hindering the development of electricity on the plate.

451. The form of machine represented in Fig. 142, is that known as Winter's. The large ball at the top of the long glass pillar on the right hand is the positive

conductor. It carries two rings, one on each side of the revolving plate, each of which is studded with points on the side next the plate. The ball on the short glass pillar at the left hand is the negative conductor. Attached

Fig. 142.



to it is the rubber, which consists of two cushions squeezing the plate between them. A piece of oiled silk covers the plate for a considerable distance from the rubbers onward, to prevent the escape of electricity before reaching the collecting points.

LEYDEN JAR.

452. The Leyden jar is a contrivance for accumulating large quantities of electricity. It usually consists of a glass jar, coated both inside and outside with tinfoil for about three-fourths of its height, while a brass rod, connected with the inner coating, projects through the mouth of the jar, and terminates in a knob.

All that is essential to the peculiar action of the Leyden jar is two opposite coatings of tinfoil, or other conducting surfaces, separated by glass or some other insulator, which must project for some distance beyond the con-

ducting surfaces, to prevent discharge taking place between them.

453. The special property of such an arrangement is, that when one of the coatings is connected with the earth, the other coating is able to receive and retain an exceedingly large quantity of electricity. The earth connection must be kept up while the charge is being given, but is not necessary afterwards.

While the charge is being given to one coating, the other acquires by induction a charge opposite in sign and nearly equal in amount, which comes to it from the earth. These two opposite charges attract one another, and their mutual attraction represses, to a large extent, the tendency of each to escape by its own self-repulsion.

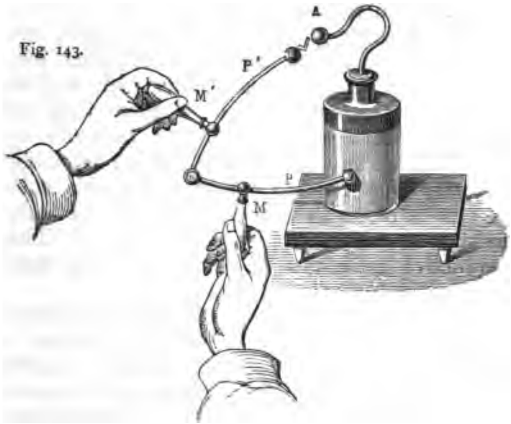
454. When the jar has been charged, if it is set upon a stool with glass legs, or is insulated in any other convenient way, it cannot be discharged by touching the knob unless the outer coating is touched at the same time. If the knuckle is applied alternately to the knob and the outer coating, a small spark will be obtained at each application; but the quantity of electricity which passes off in each spark is only an insignificant portion of the whole charge, and in a favourable atmosphere the operator may continue to draw sparks till his patience is exhausted.

To discharge the jar, a jointed discharger (Fig. 143) is commonly employed. If the jar is not insulated, but has its outer coating in connection with the earth, this coating should be touched first, and, while one knob of the discharger remains in contact with it, the other should be brought up to the knob of the jar. Just before contact, a bright spark will pass between them with a loud report.

455. If the operator holds a Leyden jar by its knob, and presents its outer coating to the prime conductor of the machine represented in Fig. 142, its outer coating will receive a positive charge from the machine, and its inner coating will receive a negative charge from the

earth, through the operator's body. When thus charged, if the operator, still holding it by the knob, were to set it down on an ordinary table, it would be discharged through

Fig. 143.



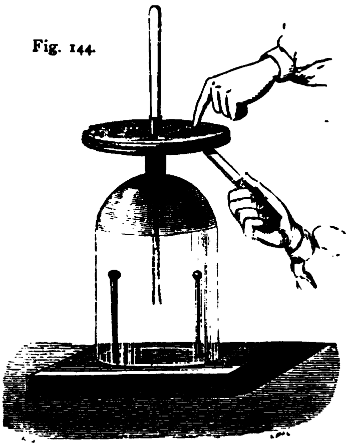
his body on coming in contact with the table. He must set it down on the stool with glass legs, and thence transfer it to the table.

456. To give a shock by a Leyden jar to a number of persons simultaneously, they must be made to join hand in hand, so as to form a chain with its two ends near together. The person who forms one end of the chain must place his hand upon the outer coating of the jar, and the person at the other end of the chain must then touch the knob. Discharge will thus take place between the two coatings through the chain of persons.

457. Capacity.—Two Leyden jars are said to have equal capacities if, when their knobs are in contact and their outer coatings connected with the earth, a charge communicated through the knobs divides itself equally between them. If it does not divide itself equally, but twice as much of it goes to one jar A as to the other B, the capacity of A is said to be double that of B.

The capacity of a jar is directly proportional to the area of either of its coatings (supposed equal and exactly opposite to each other), and inversely proportional to the distance between them (so that thin glass gives greater capacity than thick glass). It also depends, to some extent, on the nature of the glass, or other insulator, between the coatings. It is greater with any kind of solid insulator than it would be if there were only air between the coatings. The *specific inductive capacity* of an insulating substance, is estimated by comparing the capacity of a Leyden jar in which this substance is the insulator, with the capacity of an otherwise similar jar in which air is the insulator. The specific inductive capacities of nearly all solid insulators lie between the values 1·5 and 2·5.

Fig. 144.



458. Condensers.—The general name *condensers* is given to instruments constructed on the principle of the Leyden jar. Fig. 144 represents the *condensing electroscope*, which is the ordinary gold-leaf electroscope with the addition of an adjustible condenser on the top. The lower of the two plates represented in the figure is connected with the gold leaves; the upper plate (called the condensing plate) can be lifted off, and a thin coating of an insulating varnish on one or both of the plates prevents them from coming in contact.

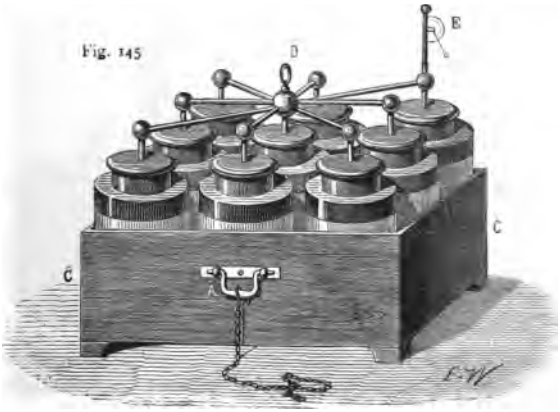
If a charge is given to the electroscope when the upper plate is away, the divergence of the leaves will diminish

as the upper plate is brought back towards its place (while kept connected with the earth), and will become inappreciably small when the two plates are as near together as the varnish allows them to come. The reason is that the capacity of the lower plate has been enormously increased, and it has therefore drawn to itself nearly the whole charge of the leaves.

On the other hand, if we begin with the upper plate in its place, and give a small charge to the lower plate, keeping the upper one connected with the earth, as represented in the figure, no indication will at first appear, but when the upper plate is lifted off the leaves will diverge.

459. **Battery of Jars.**—Fig. 145 represents an *electric* battery, or *Leyden battery*, formed by connecting the

Fig. 145



inner coatings of a number of jars, their outer coatings being in connection with the earth. Its capacity is the sum of the capacities of the separate jars. At E is shown a *quadrant-electroscope*, in which the deflection of a light arm, which carries a pith ball at its end, and hangs down vertically when unelectrified, serves to indicate in a rough way the strength of charge.

HOLTZ'S ELECTRICAL MACHINE.

460. Holtz's Machine.—The resources of the lecture table have been greatly increased of late years by the invention of Holtz's electrical machine; the general appearance of which is represented in Fig. 146.

A and B are two thin plates of glass, the former fixed, and the latter revolving when the handle of the machine

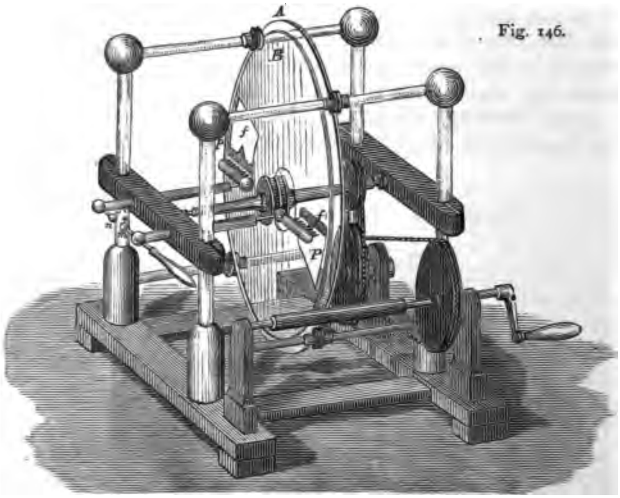


Fig. 146.

is turned. In the fixed plate there are two large openings called *windows*, near the extremities of its horizontal diameter. Adjacent to these and on the side remote from the revolving plate are glued two varnished pieces of paper called *armatures*, *f*, *f'*, each having a sharp tongue of card, projecting through the window, and pointing the opposite way to that in which the revolving plate turns. Two rows of brass points, P, P', are placed opposite the armatures, on the other side of the revolving plate, and are connected with two insulated conductors terminating

in the knobs *n*, *m*. These knobs can be set at any distance asunder. In starting the machine, they are placed in contact, and one of the armatures is electrified by holding against it an excited sheet of vulcanite, or by leading to it a wire from the conductor of a frictional machine. A peculiar *sizzling* sound is almost immediately heard, and the knobs may then be separated to a gradually increasing distance, brilliant discharge all the time taking place between them. The best machines are made double, having two revolving plates, with two fixed plates between them.

461. The explanation of the action, after the preliminary stage has been passed, is as follows:—

The two armatures *f*, *f'* are electrified, one positively and the other negatively. The positive armature (say *f'*) is opposite to that conductor which gives off positive electricity from its knob *m*, and the positive which escapes from the knob is replaced by positive which is drawn off by the points from the face of the revolving plate, an action which is favoured by the inductive influence of the positively charged armature, whether we regard it as repelling positive from the plate to the brass points or as attracting negative from the conductor through the brass points. The plate, having thus given off positive or received negative electricity, is carried round through half a revolution, and gives off negative to the other conductor through its brass points, aided by the influence of the negatively charged armature at the back.

A little electricity, just enough to replace leakage, is given off from the back of the revolving plate to the armatures, through their cardboard points, which, for this purpose, should project through the windows and nearly touch the revolving plate. It is necessary that these cardboard points should receive their supply before the brass points, otherwise they would receive either no supply or a supply of the wrong sign.

During the preliminary stage, both the armatures acquire gradually increasing charges.

462. Voss' Machine.—Holtz's machine has been greatly improved by Voss, who replenishes the charges of the armatures, not by discharge across the air to cardboard points, but by metallic contact. It is thus not necessary to employ rubbed vulcanite at starting, as the armatures usually possess an infinitesimal charge, which is rapidly multiplied by the action of the machine.

LIGHTNING.

463. Atmospheric Electricity.—Air, being a non-conductor, can be bodily charged with electricity; the law of residing only on the surface being applicable to conductors only. There is at all times electricity in the atmosphere overhead, the inductive action of which can be detected by suitable apparatus. Electricity of opposite sign resides on the surface of the earth beneath, or on the surface of objects in conducting communication with the earth; and has the greatest density on the most prominent projections.

464. Lightning is electric discharge taking place disruptively either between two portions of air or between a portion of air and the ground. Those objects are most liable to be struck by it which project furthest into the air, provided they are in free electrical communication with the earth; for these bodies are not only nearest to the thunder-clouds, but are also most strongly charged with electricity of opposite sign. Hence it is usual to protect tall buildings by means of lightning conductors, which are stout metal rods or bundles of wires, projecting somewhat above the highest part of the building to be protected, and connected at the bottom with water-pipes or gas-pipes, or in some other way brought into very perfect electrical communication with the earth.

Their upper ends should be pointed, in order that they may draw off electricity gradually rather than in violent explosions. When we wish to obtain a strong spark from the prime conductor of an electrical machine, we present a knob, and not a point.

MAGNETISM.

465. Magnets are usually either of steel or iron. Steel magnets, especially if the steel be very hard, retain their magnetization with very little change. Iron, especially if very pure and soft, is very susceptible to magnetizing influences, and may thus, while under some external influence, be more strongly magnetized than steel, but has very little power of retaining its magnetization.

466. If we take two ordinary bar-magnets, which we will call A B and C D, and present the end A to each of the ends C, D, we find it attracts one end and repels the other. If we now present the end B, we find that it repels the end which A attracted, and attracts the end which A repelled. There are thus two opposite kinds of magnetism just as there are two opposite kinds of electricity.

But there are some very important points of difference. Every magnet possesses both kinds in equal quantity, —usually confined one to each half of its length. Again, magnetism does not, like electricity, tend to escape by contact; a magnet can communicate magnetism to a piece of steel or soft iron without losing any magnetism itself.

467. To observe the communication of magnetism, take an ordinary sewing needle, and draw it four or five times across one end of a strong magnet, drawing it always in the same direction—say from point to head. The needle will be magnetized, and the end of the needle at which the rubbing commenced (in this case the point) will be similar in its magnetic properties to the end of the magnet that has been rubbed.

MAGNETIC INDUCTION.

468. If we present either end of a magnet to a piece of soft iron, we observe attraction. If the piece of iron is

small enough for the magnet to lift it easily, we shall find that when it hangs by one end from the magnet, its other end is capable of lifting by attraction a second piece of iron. This again can lift a third piece of smaller size, and so on. Each of the pieces of iron becomes, in fact, a temporary magnet. The magnetism is so distributed that dissimilar ends are in contact in each case. The end of the magnet induces properties opposite to its own in the near end of the first piece—hence the attraction between them—and properties similar to its own in the far end. An inductive action following the same law is exerted by the first piece on the second, by the second on the third, and so on. When we separate the first piece from the magnet, the magnetism which has been induced disappears almost entirely, and the chain of pieces drops asunder.

469. Contact is not essential for magnetic induction. When one end of a magnet is held near a piece of iron without touching it, the nearer side of the iron acquires, for the time, properties opposite to those of the end of the magnet which is presented. Hence, if the piece of iron is not too large, it will jump to meet the magnet and remain adhering to it. The attraction exerted upon soft iron by a steel magnet, is thus accounted for in the same way as the attraction of an insulated and uncharged pith ball by excited glass or sealing-wax. But whereas the pith ball, after touching the attracting body, is repelled, the iron, after coming in contact with the magnet, is still more strongly attracted.

470. Inductive action of the same kind, but smaller in degree, is exerted upon steel, whether previously magnetized or not. It may sometimes overpower the previous magnetization of the steel; hence similar ends of steel magnets may attract each other when brought very near, if one of the magnets is so strong as completely to overpower the other. In magnetism, as in electricity, repulsion is thus a surer test of kind than attraction.

471. If we magnetize a needle in the usual way, and

then break it in two, we find that each half is a complete magnet with ends of opposite quality. The two ends which were the ends of the whole needle, retain their original qualities, and therefore attract each other; and the two broken ends also attract each other. If we break the two half-needles, we have a repetition of the same phenomenon; indeed, if we break up the needle into any number of parts, however small, each part is a complete magnet.

Hence we infer that each particle of steel in a magnet has two sides or ends possessing opposite magnetic properties; and that in an ordinary bar-magnet, or an ordinary magnetic needle, all the sides which face one way are magnetically similar. We thus acquire the idea of magnetic polarity as a quality pervading the whole substance of a magnet, and having a definite direction, which, in an ordinary bar-magnet or magnetized needle, is the same throughout. This direction is called the direction of magnetization. In a horse-shoe magnet, it is not the same throughout (that is, not parallel to a fixed line), but bends like the magnet itself.

TERRESTRIAL MAGNETISM.

472. If we magnetize a sewing needle, and float it in water by running it through a cork, as represented in

Fig. 147.

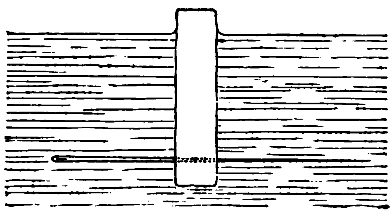


Fig. 147, we find that the needle, after a few oscillations, settles into one definite position, to which it returns if

moved away. In the ordinary compass, the same end is attained by supporting the needle on a steel point.

473. There is one definite vertical plane in which the needle tends to set itself in virtue of the external magnetic forces which act upon it; and this vertical plane is called *the magnetic meridian* at the place of observation.

If we travel along the earth's surface, always going in the direction in which the needle points, we trace out a magnetic *meridian line;* and all such lines meet in two points, called the north and south *magnetic poles*, which are some 15° or 20° distant from the geographical poles.

The student should bear in mind that the phrase "the magnetic meridian," unless accompanied by qualifying words, always means, not a line, but a vertical plane; just as "the meridian," in an astronomical sense, means a vertical plane through the place, parallel to the earth's axis. True north and south at any place are the directions of a horizontal line drawn in the astronomical meridian, and magnetic north and south are the directions of a horizontal line drawn in the magnetic meridian.

474. The angle between magnetic north and true north is called *magnetic declination.* It is said to be *west* when magnetic north is west of true north, and *east* in the opposite case.

At London it was 11° 17′ E. in 1580, zero in 1657, 24° 36′ W. in 1800, and is now something less than 20° W. Besides this large secular variation, it also varies a little according to the season of the year and the hour of the day.

475. **Dip.**—If we mount a magnetized needle on an axis passing accurately through its centre of gravity, (so that it can turn round the axis without either raising or lowering this centre,) we find that the needle does not place itself horizontally. If the axis is perpendicular to the magnetic meridian, so that the needle as it turns about always remains in the magnetic meridian, it places itself (in this country) at an inclination of nearly 70° to the horizontal, the northern end being lowest. The

direction which a needle in these circumstances assumes is called the *direction of the dipping needle*, or the *direction of terrestrial magnetic force*. It is the direction in which a magnetized needle would point if it could be suspended at its centre of gravity, with perfect freedom to turn round it all ways. The angle which the dipping needle makes with a horizontal plane is called the *dip*.

476. There is one line surrounding the earth, about midway between the two magnetic poles, at which the dip is zero. This line is called the *magnetic equator*. North of this line the north end of the needle dips, and south of it the south end dips; and in travelling from it along a magnetic meridian to either magnetic pole, the dip continually increases, till at the pole itself its value is 90° and the needle takes the vertical position.

477. There is much irregularity about the distribution of magnetic force over the earth's surface; but in a broad and general sense, the effects are nearly the same as they would be if the whole substance of the earth were magnetized in a direction parallel to the line joining the magnetic poles, with an intensity of magnetization about $\frac{1}{2000}$ of that of an ordinary steel magnet.

Since dissimilar poles attract one another, that end of a needle which points to the *north* is similar to the earth's *south* magnetic pole. To avoid ambiguity it may be called the *north-seeking pole* of the needle.

478. If a piece of steel is weighed before and after magnetization, its weight is found to be unaltered; therefore the magnetic action of the earth upon a needle does not involve attraction or repulsion of the needle as a whole to or from the earth. Also, if a magnet be used as the bob of a plumb line, the line is found to be truly vertical; hence there is no resultant force tending to carry the needle as a whole in any horizontal direction. The action is simply directive, tending to make the needle point one way, but not to give it a movement of translation. It is equivalent to two equal and opposite forces acting one on each end of the needle. These two forces

remain unchanged in magnitude and direction as the needle is turned about into different positions; and the moment of the couple (art. 21) which they exert on the needle, therefore, varies directly as the distance between their lines of action. It is zero when the needle is parallel to the line of dip, and is greatest when the needle is perpendicular to this line.

479. We can now form an idea of what is meant by *lines of magnetic force* in a uniform field.

If we consider any moderate-sized portion of space—say a space measuring several yards each way—near the earth's surface, and not near to artificial magnets nor to masses of iron or steel, we may regard it as a uniform magnetic field, because a magnetized needle will point in the same direction, and behave in all respects precisely alike, in all parts of it.

The direction which the needle tends to assume under the magnetic influences which pervade this region or *field*, is called the *direction of the lines of force* in the field; a line of magnetic force being an imaginary line drawn in the direction in which resultant magnetic force acts. As this direction is opposite for the two poles of the needle acted on, some convention is necessary to avoid ambiguity, and the convention adopted is that the direction in which a *north-seeking* pole would be urged is to be regarded as the direction of the lines of force.

480. When a magnetized needle is mounted in such a way that it can only move in a plane perpendicular to the lines of force of the field in which it is placed, it will remain in all positions in this plane indifferently. If it can only move in a plane oblique to the lines of force, it will assume that direction which makes the smallest angle with the lines of force. If its movements lie in a plane parallel to the lines of force, it will assume the direction of these lines.

481. Without special apparatus, experiments on dip, even of a rough kind, are difficult of execution; for if a needle turns about an axis which does not pass exactly

through its centre of gravity, or turns about a point which is not exactly coincident with its centre of gravity, the weight of the needle tends to make it assume one definite inclination to the horizon; and the inclination actually assumed will be intermediate between this inclination and the true magnetic dip. In rough experiments, there is danger of the magnetic effect being completely masked by the gravitation effect.

482. The force which the earth, as a magnet, exerts upon either pole of any magnet at its surface, can be resolved into two components, one horizontal and the other vertical. When the dip exceeds 45° (as it does in this country) the vertical component is the greater of the two.

483. Pieces of soft iron are rendered temporarily magnetic by the inductive action of the earth's magnetism. If an iron rod, or a poker, is held parallel to the dipping needle, the lower end of the rod becomes for the time a north-seeking pole, and its upper end a south-seeking pole, as is at once seen by bringing a suspended magnetic needle near its ends. A similar result, but not quite so powerful, is obtained by holding the rod vertically, the effect being in this case due to the vertical component of the total force. If it is held horizontally in the direction in which a compass needle points, the effect is still feebler, because it is due to the horizontal component only.

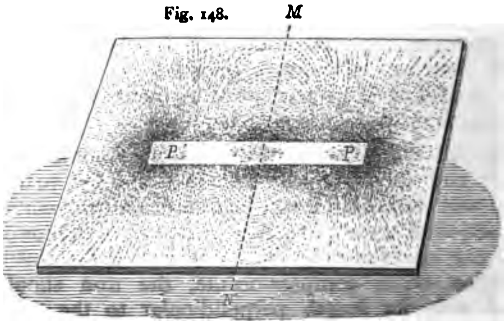
484. All iron, however soft, possesses some retentive power; and fire-irons which have stood for years in a nearly vertical position, will be found, on examination, to have acquired some permanent magnetism, the lower end being the north-seeking end. In testing for this effect by action on a compass-needle, the iron should be placed with its length perpendicular to the magnetic meridian, so as to avoid disturbance from temporary induction.

GENERAL THEORY OF MAGNETISM.

485. An artificial magnet produces, like the earth, a magnetic field all round it, and it is easy to trace approximately the lines of force which pervade this field. A small magnetized needle, hung by a thread tied to its centre, turns into a nearly vertical position when held over one pole of the magnet (which we will suppose to be a bar-magnet laid flat upon a table), thus showing that the lines of force in the region immediately over the pole run nearly vertically. If held over the other pole, it also places itself vertically, but with ends reversed. If held over the middle of the magnet, it places itself horizontally, because the lines of force are here horizontal. A line of force may be approximately traced from one pole to the other by carrying the needle always in the direction in which it points. A short piece of soft iron wire may be used instead of a needle, and will give similar indications; for it becomes temporarily magnetized in the direction of its length by the inductive action of the magnet.

486. The forms of the lines of force in the plane of the table may be shown by means of wrought-iron filings.

Fig. 148.



On sifting them over the table in the neighbourhood of the magnet, and then tapping the table, they will be seen

to arrange themselves (as shown in Fig. 148) in a system of curves, which are lines of force. The filings in any one of these curves are temporary magnets, with contiguous poles dissimilar, and therefore attractive.

487. If we continue to rap the table, we observe that all the lines of filings move in towards the magnet; and the individual filings for the most part do not travel along the lines of force, but deviate towards the concave side of the curves to which they belong. The rule for a small spherical piece of soft iron in a magnetic field is, that it tends to move in the direction in which magnetic force increases most rapidly, a direction which is often very different from that of the lines of force themselves.

488. By the employment of very powerful magnets—usually electro-magnets, which we shall describe further on—it has been shown that nearly all substances, whether solid, liquid, or gaseous, are affected by magnetic forces. Some substances, especially nickel, show behaviour similar to that of iron. An elongated piece of nickel places itself parallel to the lines of force; and a spherical piece moves from places of weaker to places of stronger force. Such substances are called *ferro-magnetic* or *paramagnetic*. Other substances, notably bismuth, behave differently. An elongated piece of bismuth places itself across the line joining two opposite magnetic poles, and a spherical piece moves from places of stronger to places of weaker force. Such substances are called *diamagnetic*.

489. This difference of behaviour arises from a difference in inductive action. The direction of temporary magnetization of a piece of bismuth placed in a magnetic field is opposite (end for end) to that of a piece of nickel or iron similarly placed. An iron needle laid along lines of force is pulled outwards at both ends by magnetic attractions, and is therefore in stable equilibrium; a bismuth needle held in the line joining two opposite poles has both its ends repelled, and is therefore in unstable equilibrium. A ball of iron held near a pole of a magnet is attracted because its near side becomes a dissimilar

pole. A ball of bismuth is repelled because its near side becomes a similar pole.

490. Two Imaginary Magnetic Fluids.—In accurate discussions respecting the forces exhibited by magnets, it is necessary to introduce the conception of two fluids of opposite quality residing respectively in the two halves of an ordinary magnet. One may be called the north-seeking and the other the south-seeking fluid. Each fluid repels its own kind and attracts its opposite. The student is not to regard these two fluids as real substances, but as convenient fictions.

They are distributed very sparingly over the central parts of an ordinary magnet, but very thickly near the ends; and the quantity of north-seeking fluid which resides in one half of the magnet, is precisely equal to the quantity of south-seeking fluid in the other half. Every permanent magnet contains equal quantities of the two fluids. The unit quantity is that which, if collected at a point, would attract or repel with unit force an equal quantity collected at another point at unit distance; and the force varies inversely as the square of the distance.

CURRENT ELECTRICITY.

VOLTAIC BATTERY.

491. When two metals are partially immersed in a liquid which acts chemically upon one of them, and are not allowed to touch each other within the liquid, but are connected by a wire outside of it, an electrical current flows through the wire, and continues to flow as long as the chemical action goes on. Such an arrangement constitutes a *galvanic* or *voltaic cell;* and several such cells connected so as to form a series constitute a *galvanic* or *voltaic battery.* To form the series, one of the metals in the first cell is connected by a short thick wire with the dissimilar metal of the second cell; the other metal of the second cell is in like manner connected with the dissimilar metal of the third cell, and so on. There will thus be one metal in the first cell and one in the last remaining unconnected. These, or wires leading from them, are called the *terminals* or *electrodes* or *poles* of the battery.

492. In all the batteries in ordinary use, zinc is one of the two metals employed. The other metal is frequently copper. In Grove's battery it is platinum. In Bunsen's and some other batteries, a very hard form of carbon, which is deposited in the retorts at gas-works, is employed instead of a second metal. Dilute sulphuric acid is usually employed as the liquid to act upon the zinc. In some of the best forms of battery, there is a second fluid in contact with the other metal; the two fluids being prevented from free admixture by a porous partition.

493. Direction of the Current.—When the two wires which form the terminals of the battery are kept separate from each other, and at the same time unconnected with the earth, very little chemical action goes on in the battery. When they are connected, we have a vol-

taic circuit, round which a current steadily flows. Positive electricity flows through each cell from the zinc to the other metal, and completes its circuit by flowing through the terminal wires from the last copper, platinum, or carbon, to the first zinc. Or we may state the same fact by saying that negative electricity flows through the terminal wires from zinc to copper, and through each cell from copper to zinc. There is no distinction between a positive current flowing in one direction and a negative current flowing in the opposite direction; the two expressions are to be regarded as different statements of the same fact.

By the general consent of all writers on electricity, the direction of the positive current is called simply *the direction of the current*,—not from any idea that positive or vitreous electricity is more real than resinous or negative electricity, but simply for the sake of brevity. The direction of the current of a battery, then, in the external portion of the circuit, is from the copper, platinum, or carbon terminal to the zinc terminal. The terminal which gives off positive electricity—that is, the copper, platinum, or carbon terminal—is called positive; and the terminal which gives off negative electricity (the zinc terminal) is called negative.

494. Relation between Frictional and Voltaic Electricity.—Electric currents also occur in connection with frictional electricity. In discharging a Leyden-jar, we establish a current of brief duration between the two coatings; and if we lead a wire from the conductor of an electrical machine either to the earth or to the rubber, a tolerably steady current flows through this wire when the machine is turned.

Such currents, though the same in kind, are exceedingly different in degree from the current of a galvanic battery. The frictional machine is like a steam-engine with a very small boiler containing steam at enormously high pressure, the galvanic battery is like an engine with an enormous boiler containing steam at very low pressure.

The battery gives an enormously larger quantity of electricity than the machine, but the electricity which it furnishes has but little power of overcoming resistance. Pure water, for example, is practically a non-conductor for galvanic electricity, although the slightest film of moisture destroys insulation for frictional electricity. This relative feebleness of galvanic electricity is technically expressed by saying that the *electromotive force* of a galvanic battery is much less than that of a frictional machine.

495. **Effects of Currents.**—An electric current produces heat in the wires through which it flows. Short and thin wires of iron or platinum are easily heated red-hot, or even melted, by the current from an ordinary battery of four or five cells.

496. A horizontally-balanced magnetic needle, like that of an ordinary compass, is deflected out of the magnetic meridian, if a current nearly parallel to the needle is sent over it or under it at a small distance. The direction of deflection is reversed by reversing the direction of the current, and is also reversed by making the current pass below instead of above the needle. This is the principle of the galvanometer.

497. If a current is sent through a saline solution, or through acidulated water, the liquid is decomposed; one constituent being deposited or given off at the place where the current enters the liquid, and the complementary constituent at the place where the current leaves the liquid. This operation is called *electrolysis*, and the liquid which is decomposed is called the *electrolyte*.

498. **Principles of Measurement of Currents.**—A current may be measured either by the couple which it exerts upon a given magnetic needle in a given position with respect to the wire which conveys the current, or by the quantity of a given electrolyte which it decomposes in a given time. These two modes of measurement give strictly proportional results; and by employing powerful frictional machines to produce currents, it has been shown that the quantity of electricity which passes

in a given time is proportional to the current as measured in either of these ways. The numerical comparison of currents is therefore possible without any ambiguity or convention. The numerical value of a current is usually called the *strength* of the current.

CONDITIONS WHICH DETERMINE STRENGTH OF CURRENT.

499. Ohm's Law.—The current sent by a battery of several cells through a short and thick wire of good conducting material connecting its two extreme plates, is not sensibly stronger than the current which would be sent through the same wire by one of the cells alone. The reason is that the resistance which such a wire opposes to the passage of a current is insignificant in comparison with the resistances in the cells themselves, and the advantage which the battery possesses over the cell as regards electromotive force is exactly counter-balanced by its greater resistance. On the other hand, the current which a battery of n cells will send through a wire of such length and character as to have resistance very much greater than that of the battery, is n times as strong as the current which one cell would send through the same wire; for the resistances of the battery and of the cell may in this case be neglected, and the ratio of the currents will be the ratio of the electromotive forces.

500. The general law—called *Ohm's law*—is that *the strength of a current varies directly as the electromotive force which maintains it, and inversely as the resistance which it traverses.* The electromotive force depends only on the battery, and is n times as great for a battery of n cells arranged in a series as for one cell. The resistance is the sum of two parts, one being the resistance in the connecting wire, and the other the resistance in the battery. The resistance in the wire (often called the external resistance) is independent of the battery, but the resistance in the battery (often called the internal resistance) is n times as great for n cells in series as for one. In these statements we suppose all the cells to be alike.

501. Resistance of Wires.—The resistance of wire of a given size and description varies simply as its length. One cell will give the same current through a given length of wire, that two cells will give through a double length of the same wire.

The resistances of wires of the same length and material are inversely as their cross-sections; so that, if we denote length by l and sectional area by s, two wires of the same material will have equal resistances when the values of $\frac{l}{s}$ are the same for both. The direct test of equality of resistance is that the substitution of one wire for the other in a circuit makes no difference in the strength of current. The resistances of any two wires of the same material are directly as the values of $\frac{l}{s}$.

The complete expression for the resistance of a wire is $\rho \frac{l}{s}$, ρ being a factor depending on the nature and condition of the material of which the wire is composed, and called the *specific resistance* of the material.

502. Specific Resistance.—Gold, silver, and copper are the metals of smallest specific resistance. This is equivalent to saying that they are the best conductors. The specific resistance of copper when very pure is only about one-sixth of that of iron; in other words, the electric conductivity of copper is about six times that of iron. This is nearly the same as the ratio of their thermal conductivities, and in general the electrical and thermal conductivities of metals are nearly proportional. Both kinds of conductivity—in the case of metals—decrease with increase of temperature.

503. Resistance of Battery.—The resistance of a battery-cell is mainly the resistance of the liquid or liquids which it contains, and of the porous partition, if there be one. These may be regarded as composing one conductor, very short and of large sectional area, as compared with a few yards of wire, but of very much greater specific resistance. Accordingly, a few yards of wire may be

equal in resistance to one cell of a battery. The resistance of a battery of n cells in series is n times that of one cell, on the same principle that the resistance of n yards of a given kind of wire is n times that of one yard.

504. To make an experimental comparison of the resistance of a battery with that of a given kind of wire, we may (theoretically at least) proceed as follows:—

Measure the current obtained when the terminal plates of the battery are connected by a piece of the wire of measured length l_1. Then increase the length of wire till the current is reduced to half its former strength; and let l_2 be the length of wire now in circuit. The total resistance in circuit will now be double what it was before; so that, if b denote the resistance of the battery, expressed in length of wire, we have

$$l_2 + b = 2\ (l_1 + b),$$

whence

$$b = l_2 - 2\ l_1.$$

505. **Divided Circuit.**—If the poles of a battery, instead of being connected by a single wire, are connected by two wires, which form independent courses, A C B, A D B, the resistance is less than it would be with one of these wires alone. If the two wires are in all respects equal, they are together equivalent to a single wire of half the resistance of one of them; we may, in fact, regard them as a single wire of double cross-section.

The general rule for such cases is best given as follows:— Let r denote the resistance of a wire, and let $\frac{1}{r}$ be called its *conducting power*. Then, if there are two or more wires which furnish independent paths between the same two points A, B, the joint conducting power is the sum of the separate conducting powers. For example, if the resistances in A C B and A D B are represented by the numbers 2 and 3, and x denote their joint resistance, we have

$$\frac{1}{x} = \frac{1}{2} + \frac{1}{3};\ \text{whence } x = 1\tfrac{1}{5}.$$

THERMAL EFFECTS.

506. **Generation of Heat by Currents.**—The law which governs the generation of heat by currents, has been investigated by sending a current through a coil of fine platinum wire immersed in cold water, and observing the increase of temperature of the water in a measured time. It has thus been found that, for a given wire maintained at a nearly constant temperature, the heat produced in a given time varies as the square of the current; and that, when different wires are traversed by the same current, or by equal currents, the quantities of heat generated in them in a given time are directly as their resistances. If we multiply the resistance of a wire by the square of the current which passes through it, we obtain a measure of the heat generated in the wire in each unit of time.

507. When a current is sent through a wire in air, the rate at which it begins to rise in temperature will depend partly on the rate at which heat is generated in it, and partly upon its thermal capacity. A fine wire will rise in temperature faster than a thick one, for a double reason;—there is more heat generated in it per unit of length, and it has also less thermal capacity per unit of length. Hence a wire of the thickness of a horse-hair is often fused, before a stout wire in the same circuit has exhibited any noticeable elevation of temperature.

508. In the heating of a wire forming part of a voltaic circuit, we have some interesting examples of the transformation of energy. The potential energy of chemical affinity runs down in the cells, producing an equivalent amount of energy in the shape of electricity in motion; and the energy of the current is in its turn converted into heat in all parts of the circuit. The upshot of these transformations is, that heat is generated precisely equivalent to the chemical action that goes on; but instead of being confined to the cells where the action takes place, it is divided between the battery and the connecting wire, in direct proportion to their resistances.

ELECTROLYSIS.

509. Consumption of Zinc in Battery.—In batteries which are in proper working order, the quantity of zinc dissolved in each cell in a given time varies directly as the strength of the current; and this is true not only in comparing the same battery with itself when different resistances are introduced into its circuit, but also in comparing one battery with another of a different kind.

Remembering that the quantity of electricity conveyed by a current in any time is equal to the strength of the current multiplied by the time, we may put the above statement in the following form:—*The quantity of zinc dissolved in one cell is a measure of the quantity of electricity that passes.*

If we increase the number of cells in a battery, and thus increase the strength of the current; not only will the total quantity of zinc dissolved in a given time be increased, but the quantity dissolved *in each cell* will be increased in the same ratio as the current.

510. Amount of Electrolytic Action.—In like manner, when, by means of a battery, we effect electrolysis either in a single cell or in a series of cells, the amount of decomposition effected in any one of these cells is a measure of the quantity of electricity that passes.

If the electrolytic cells contain a solution of potassium chloride, then, for every 65 parts (by mass) of zinc dissolved in one cell of the battery, there will be 71 parts of chlorine and 78 of potassium separated in each electrolytic cell.

If the liquid in one of the electrolytic cells is a solution of zinc sulphate, the quantity of zinc separated in this cell will be equal to the quantity of zinc dissolved in a cell of the battery.

511. General Law of Equivalence.—The general law is, that the quantity of any substance separated in an electrolytic cell is chemically—or rather electro-chemically—equivalent to the quantity of zinc dissolved in each

battery-cell. The same substance in different states of combination may have different electro-chemical equivalents. For example, in decomposing ferrous chloride, 56 parts of iron are separated from 71 of chlorine for every 65 parts of zinc dissolved, so that here we have 56 parts of iron equivalent to 65 of zinc; but in decomposing ferric chloride, $37\frac{1}{3}$ parts of iron (two-thirds of the former quantity) are separated from 71 of chlorine for every 65 parts of zinc dissolved, so that here $37\frac{1}{3}$ parts of iron are equivalent to 65 parts of zinc. In both cases 71 parts of chlorine are equivalent to 65 of zinc.

512. Negative Electromotive Force.—An electrolytic cell is to be regarded as a reversed battery-cell—that is, as a battery-cell whose connections are reversed, so that the current traverses it in the wrong direction. If we actually reverse in this way the connections of one cell of a battery, so that the current traverses it from copper to zinc instead of from zinc to copper, it will cancel the electromotive force of one of the other cells; and in like manner every electrolytic cell contributes negative electromotive force to the circuit. The number of battery-cells required to work a given series of electrolytic cells, is determined by the condition that the total electromotive force in circuit must be positive.

513. Electromotive Force Related to Heat of Combination.—The electromotive force of a cell is proportional to the heat of chemical combination due to the total chemical action which takes place in it during the consumption of a given quantity of zinc. The amount of this heat is very different for different batteries, being nearly twice as great for a Grove's cell as for a Daniell's.

514. Most Economic Electrolysis.—Remembering that the quantity of zinc dissolved is a measure of the quantity of electricity that passes, we may state the law thus:—The electromotive force of a cell is proportional to the thermal value of the chemical action which takes place in the cell during the passage of a given quantity of electricity. In like manner the loss of electromotive force,

caused by the presence of a decomposing cell in a circuit, is represented by the heat which would be generated by a reversal of the chemical action which takes place in it. The potential energy of chemical affinity which runs down in a battery-cell, is equal to the electromotive force of the cell, multiplied by the quantity of electricity that passes; and the potential energy developed in an electrolytic cell, is equal to the negative electromotive force of the cell, multiplied by the quantity of electricity that passes.

Since there must always be more positive than negative electromotive force in circuit, the potential energy which runs down is always in excess of that which is developed. This excess is transformed into heat, which is distributed over the different parts of the circuit in proportion to their resistances.

In order to produce a given amount of electrolytic effect, with the least possible consumption of material in the battery, the number of battery-cells employed should be the smallest that is sufficient to maintain a current.

515. Voltameter.—An electrolytic cell containing water acidulated with sulphuric acid gives off oxygen[1] and hydrogen. Such a cell, provided with means for collecting and measuring the gases evolved, is called a voltameter. The quantity of gas evolved is a measure of the quantity of electricity that passes; and the quantity of gas evolved *in a given time* is a measure of the strength of the current.

516. Anions and Cations.—Electrolysis always consists in the separation of a substance into two constituents, one of which is separated at the place where the current enters the solution, and the other at the place where the current leaves it. In the intermediate region no effect is observed. The two constituents must there-

[1] The electrodes of the cell must be of platinum. If they are of copper, oxygen will not be given off, but will enter into combination with the copper.

fore travel in opposite directions, in some invisible manner, through the liquid. They are therefore called *ions* (ιων, going). That which travels with the current, and is separated at the place where the current leaves the liquid, is called the *cation* (κατιων, going down), because it may be said to run down the stream; that which travels against the current, and is separated at the place where the current enters, is called the *anion* (ανιων, going up). Hydrogen and the metals are cations; oxygen and chlorine are anions.

CONTACT ELECTRICITY AND POLARIZATION OF ELECTRODES.

517. **Contact of Metals.**—Volta, the inventor of the galvanic battery, attributed the origin of its electricity to the contact of dissimilar metals.

Sir Wm. Thomson has shown, by a very delicately arranged apparatus, that, when a piece of zinc is soldered to a piece of copper, a positively electrified body suspended over the junction is urged from the zinc towards the copper, and a negatively electrified body is urged from the copper towards the zinc; the zinc thus appearing to be positively and the copper negatively electrified.

He has also found that, when copper filings are allowed to run through a zinc funnel connected with the earth, and are caught in an insulated vessel, this vessel becomes first slightly and then by degrees strongly negative. It seems that any two metals, and probably any two substances whatever, become slightly electrified by touching each other.

518. **Polarization in Batteries.**—In the simpler forms of galvanic cell, such as a plate of zinc and a plate of copper in dilute sulphuric acid, there is a continual evolution of hydrogen at the copper plate, while an equivalent quantity of oxygen enters into combination with the zinc plate, and goes to form zinc sulphate. Some of the hydrogen thus evolved adheres to the copper plate, and produces a reverse electromotive force. This action, which

is the principal cause of the rapid weakening of the current in batteries composed of such cells, is called *polarization.*

519. Secondary Batteries, or Accumulators.—A reverse electromotive force is in like manner called out by electrolysis, and this fact is utilized in the construction of what are called *secondary* batteries. Any cell which is first subjected to electrolysis by a current from an external source, and then gives of itself a current in the opposite direction, is called a *secondary cell*, and a combination of such cells is a *secondary battery.* Recent improvements in their construction have called public attention to them as an important means of storing up energy, to be used when and where it is wanted. Hence they have received the name of *storage batteries* or *accumulators.* Faure's accumulator consists of two leaden plates of large surface, covered with minium (red oxide of lead), rolled up together with flannel between them, and immersed in dilute sulphuric acid. The primary current, which is usually supplied by a dynamo machine (art. 544), changes a portion of the minium on the positive plate into peroxide of lead, and reduces a portion of the minium on the negative plate to metallic lead in a spongy condition. A battery of these cells freshly charged by a dynamo machine will give a powerful current for some hours; and an interval of several hours between charging and using does not involve much loss. Planté had previously constructed secondary cells consisting in like manner of two lead plates spirally coiled in dilute sulphuric acid, and had greatly improved their action by coating them with peroxide, which he did, not by mechanical means, but by sending through the cell a succession of currents in opposite directions, with intervals of rest between.

520. Two-fluid Batteries.—In some of the best forms of battery, polarization is avoided, or at least reduced to a minimum, by employing two liquids separated either by a membrane or by a porous partition of unglazed earthenware. The liquid in contact with the zinc plate is dilute sulphuric acid, and the other liquid (which

is in contact with the other plate) is selected for its power of taking up hydrogen. In Daniell's battery, it is a saturated solution of sulphate of copper, the second plate being copper. In Grove's and Bunsen's batteries it is strong nitric acid, the second plate being platinum in the former battery and carbon in the latter. These three forms of battery, especially Daniell's, are very constant in their action.

521. **Amalgamation of Zinc Plate.**—It is usual to amalgamate the zinc plates of a battery (by washing them with acid and then rubbing them with mercury). The reason is, that when ordinary commercial zinc is used without amalgamation, local currents are formed between different portions of the same plate, owing to inequalities or impurities. This *local action*, as it is called, is wasteful, and does not aid, but in fact weakens, the current in the general circuit. Amalgamation renders the surface uniform, and prevents this injurious action. Amalgamation, however, should not be employed when the greatest possible constancy is desired.

ELECTRO-METALLURGY.

522. The copper plate in a Daniell's cell receives a continual deposit of fresh copper from the solution of sulphate of copper in which it is immersed, and it was this circumstance which first suggested the idea of *electro-metallurgy*, or the electric deposition of metals, which now occupies an important place in the arts.

Electro-metallurgy includes two distinct kinds of operations, according as the deposit is intended to form a permanent coating (as in electroplating), or to yield copies of an original (as in electrotype). For the latter purpose, a mould in some soft material, such as gutta-percha, is first taken from the original, and this mould, after being faced with plumbago (which is a conductor), is made to do duty for the copper plate of a Daniell's cell, until a sufficiently thick layer of copper has been deposited upon it. The sheet of copper thus deposited is then separated from the mould, of which it is a perfect impression.

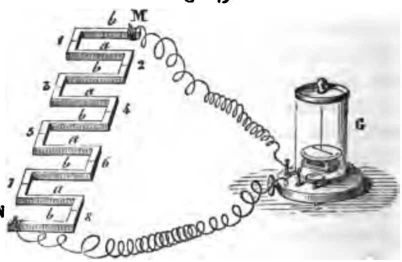
THERMO-ELECTRICITY.

523. Balance of Electromotive Forces in Circuit of Metals.—If two, three, or more metals, all at the same temperature, are joined together so as to form a closed circuit, no current passes; the electromotive forces due to the contacts of dissimilar metals exactly balance each other. This balance, however, is destroyed if some of the junctions are made warmer than others. The currents thus produced by differences of temperature in a metallic circuit are called *thermo-electric.*

524. Thermopile.—The *thermopile* devised by Nobili is a kind of thermometer based on this principle. It contains a number of small bars of antimony and bismuth arranged alternately in one continuous series, the two extreme bars being connected with binding screws, from which wires can be led to a delicate galvanometer. The direction of the current is from bismuth to antimony through hot junctions, and from antimony to bismuth through cold junctions,—the arrangements being such that hot and cold junctions succeed each other alternately.

Fig. 149 shows the principle of the arrangement. The

Fig. 149.



bars marked *a* are antimony, those marked *b* bismuth. The junctions 1, 3, 5, 7 are to be at one temperature, and the junctions 2, 4, 6, 8 at a different temperature.

In the thermopile, as actually constructed, the bars are built up into a compact mass, as in Fig. 150, the odd junctions being at one end and the even junctions at the other, and are mounted as shown at C D, Fig. 151. P P′ are the binding-screws, connected respectively with the two extreme bars.

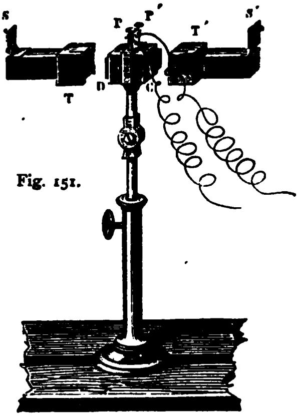
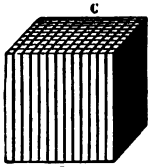


Fig. 151.

Fig. 150.



S T and S′ T′ are movable covers, by the help of which the two ends of the pile can be either screened or exposed.

525. The electromotive force of a thermopile varies directly as the difference of the temperature of its ends, so long as these lie within the limits of ordinary atmospheric temperature. It is also directly proportional to the number of junctions. The instrument can be made extremely sensitive, and has been especially employed for observations on radiation.

526. Mure and Clamond's Pile.—A quite different class of thermopiles have lately been introduced, intended not for thermometric purposes but for taking the place of a small battery. In one of the most successful of these, the two substances employed are iron and an alloy of zinc and antimony, and the pile has the form of a hollow cylinder. The junctions which abut on the internal space are heated by means of a gas flame, while the external junctions are kept cool by contact with the

air. Such piles have been used commercially for the purposes of electro-metallurgy.

527. **Neutral Point.**—For every two metals, there is a particular temperature called their *neutral point*, such that a thermopile constructed of these metals will give no current when one end is just as much above the neutral point as the other is below it.[1] If one end is kept at a constant temperature lower than the neutral point, while the other, initially at the same temperature, is steadily raised, the current will first increase to a maximum, then diminish to zero (becoming zero when the two temperatures have the neutral point for their arithmetical mean), and will then increase with opposite sign.

These changes can be shown with copper and iron wire. Let a piece of iron wire, A B, be joined to copper wires at both ends, and let the copper wires be led to a delicate galvanometer. By gently heating one of the junctions A or B, a current will be produced which will deflect the needle in one direction; but if the heating is continued to redness, the needle comes back, and is still more strongly deflected in the opposite direction.

ELECTRO-MAGNETISM.

528. **Rule for Deflection of Needle.**—We have already stated that a magnetized needle can be deflected from the meridian by an electric current. The following rule—known as Ampère's rule—gives the direction of the deflection. Imagine yourself identified with the wire through which the current is passing, *the current being supposed to enter at your feet and come out at your head;* then when you look towards the needle, you see the *north-seeking pole deflected to your left*, and therefore the south-seeking pole to your right. It follows from this rule that

[1] This will serve as a general definition of the neutral point of two metals. In certain special cases it is necessary to add the condition that the two temperatures must be taken very near together.

if we coil the wire into the form of a ring, and place the needle in its centre, all parts of the wire will conspire to deflect the needle the same way.

529. **Galvanometer.**—The galvanometer (one form of which is shown in Fig. 152) is an instrument constructed on this principle. A wire is carried several times round a ring (the different convolutions being insulated from each other by winding silk round the wire), and the two ends of the wire are connected with binding-screws, for making connection with the battery or other apparatus which is to furnish the current. The needle is suspended in a horizontal position in the centre, either by balancing it on a point or hanging it from a fine thread or fibre,—this latter mode of suspension being the more delicate of the two. If the current is allowed to flow steadily, the needle will, after more or less oscillation, assume a position of equilibrium, which may differ by anything less than 90° from its undeflected position. The earth's magnetic action tends to bring the needle back into the plane of the coil; the action of the current tends to set the needle at right angles to this plane; and it will take some position intermediate between these two, dependent on their relative strength.

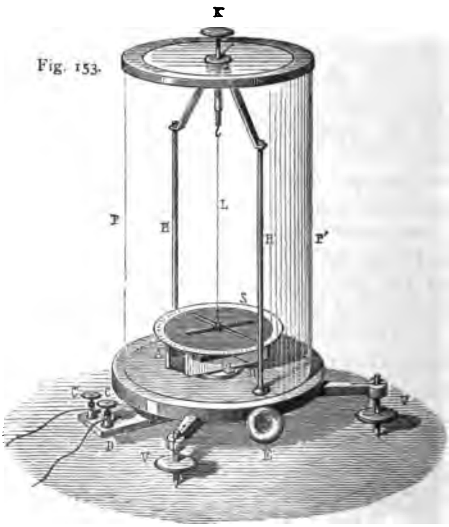
Fig 152.

530. In the *tangent galvanometer*,—the needle of which is short, and is attached to a long pointer,—the strength of the current is inferred from the deflection thus obtained; the current being proportional to the "tangent" of this deflection.

531. In the *sine galvanometer* a larger deflection is obtained (with the same coil and current), partly because the needle is long, and partly because the plane of the coil is turned after the needle till it overtakes it, so that the coil is in the position in which the deflecting couple which it exerts upon the needle is the greatest possible. The graduated circle shown in the lower part of Fig. 152 is for indicating the angle through which the coil has been turned,—which is the same as the angle through which the needle has been deflected. The current is proportional to the "sine" of this angle, and must not exceed a certain limiting strength, or the coil will never overtake the needle.

The graduated circle at the level of the needle is for

Fig. 153.



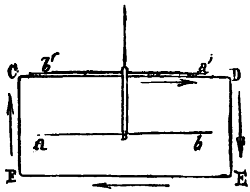
indicating the deflection obtained when the instrument is used as a tangent galvanometer.

532. Sometimes a galvanometer has two pairs of binding-screws, connected with two distinct coils, wound on the same ring, and as nearly as possible equal in all respects. It is then called a *differential galvanometer*, and can be used for directly measuring the difference of two currents, one being sent through each coil, in opposite directions.

533. **Astatic Galvanometer.**—The astatic galvanometer (Fig. 153) is much more sensitive than either the tangent or sine galvanometer, but its indications are not so easily reduced to proportional measure.

It has two needles *a′ b′*, *a b* (Fig. 154) as nearly equal as possible, fastened to one upright stem, *with their poles pointing opposite ways.* The directive actions of the earth on the two needles are opposite, and hence the resultant directive action of the earth on the two combined is very small. The coil of the galvanometer, on the other hand, is so placed that the current tends to deflect both needles the same way. The coil, which is shown in section in Fig. 154, is approximately rectangular, its longest dimension being horizontal. One of the needles *a′ b′* is just above, and the other *a b* is below the upper part of the coil. The current in this part of the coil would urge them opposite ways if their poles were similarly directed, and hence, as their poles are oppositely directed, it urges them the same way. The current in the lower part and ends of the coil assists in deflecting the lower needle, and is too distant from the upper needle to have much effect upon it. The coil is thus placed in a position of great advantage as compared with the earth, and the deflection is proportionately large.

Fig. 154.



534. **Quantitative Laws.**—The action of a current upon a needle may be regarded as consisting of two

forces, one on each pole. The force exerted by a straight portion A B of a current upon a pole C is perpendicular to the plane A B C, and the force exerted by a circular current (or by any plane current) upon a pole in its own plane is perpendicular to this plane. When the pole is in the centre of the circle the force is proportional to the following quantities:—

1. The strength of the pole, as measured by the attraction or repulsion which it would exert upon a given pole at a given distance.

2. The strength of the current.

3. The length of wire through which the current flows. If the wire (for instance) is carried ten times round the circle, the length will be ten circumferences.

4. The inverse square of the distance of the wire from the pole; that is to say, the inverse square of the radius of the circle.

535. **Forces Mutual.**—The forces between a current and a magnet are, like all other forces in nature, mutual, reaction being equal and opposite to action. The coil of a galvanometer is acted on by a couple equal and opposite to that which deflects the needle.

MAGNETO-ELECTRICITY

536. **Induced Currents.**—Besides this mechanical action, tending to move the coil, and depending on the position of the needle, there is another effect (not mechanical) due to the motion of the needle. The current is weakened or strengthened by the motion of the needle; weakened when the needle moves in the direction in which the current urges it; strengthened when the needle moves in the opposite direction.

More generally, when a magnet, and a conducting wire forming part of a closed circuit, are moved, one or both, in such directions that the mutual forces between them do positive work, the current is weakened; and if the motion is such that these forces do negative work (being

a motion which, on the whole, is resisted by them), the current is strengthened (see art. 35). The effect, in both cases, is merely transient; so that, as soon as the motion ceases, the current returns to its normal strength, which is not affected by the presence of magnets at rest.

537. Again, if there is a closed circuit (for instance, a coil of wire with its ends joined) through which no current is passing, the motion of a magnet in its neighbourhood will induce a current in it, the direction of this current being always such as to oppose the motion. It matters not which of the two bodies (the circuit and the magnet) is moved. The effect is wholly dependent on their relative motion.

These are the fundamental principles of magneto-electricity—that branch of electricity which treats of the excitement of currents by means of magnets. Currents so excited are said to be *induced.*

538. **Electro-magnets.**—We have just seen that a current can be produced by means of a magnet. On the other hand, a piece of iron can be magnetized by means of a current.

Let wire, covered with silk or other insulating material, be coiled round a bar of iron, like thread upon a reel, and let a current be sent through the coil. The iron will, for the time, become a magnet, its north-seeking pole being at that end towards which a north-seeking pole at the centre of the coil would be urged by the action of the current.

If the current is stopped, the magnetism of the iron disappears almost entirely; and if the current is reversed, the magnetism of the iron is reversed, end for end.

A piece of iron thus surrounded by a coil is called an *electro-magnet.* The iron itself is called the *core* of the electro-magnet.

An electro-magnet is *made* by sending a current through its coil; *unmade* by stopping the current.

The purest and softest iron makes the strongest electro-magnets. If steel be used instead of iron, it makes a

weaker electro-magnet, but retains a large share of its magnetism after the cessation of the current. This is one of the commonest ways of making strong permanent magnets.

539. **Ruhmkorff's Coil.**—The *induction coil*, or *Ruhmkorff's coil*, is an instrument whose action depends partly on the induction of magnetism by a current, and partly on the induction of currents by a magnet.

In the first place there is an electro-magnet, consisting of a soft iron core encircled by a coil of stout copper wire called the *primary coil*. Through this coil a current is sent, —not continuously, but with a rapid succession of makes and breaks, so that the electro-magnet is alternately made and unmade. Round the electro-magnet is wound another coil, consisting of a much greater length—often many miles—of much finer copper wire, very carefully insulated. This is called the *secondary coil*. The making of the electro-magnet induces a current in one direction in the secondary coil, and the unmaking induces a current in the opposite direction. The making of a magnet within a coil has, in fact, the same effect as bringing up the magnet from a great distance and thrusting it into the coil, and the unmaking has the same effect as removing it again. Owing to the great number of convolutions of the secondary coil, each of which contributes its own portion of electromotive force to the secondary circuit, the total electromotive force in this circuit is very great, quite comparable with that of the electricity yielded by a frictional machine. A rapid succession of powerful sparks passes between the terminals of the secondary coil, at a distance varying, in the case of small machines, from a quarter of an inch to two inches, and in large machines sometimes reaching 18 inches or 2 feet.

540. The making and breaking of the primary circuit is usually effected automatically. The electro-magnet, as soon as it is made, attracts to itself a small movable piece of iron called the hammer, causing it to leave another fixed piece called the anvil, and thus breaking the circuit. The magnet being thus unmade, the hammer (which is

constantly urged towards the anvil by a spring) is released, and by striking the anvil closes the circuit again. The hammer is thus kept in a state of rapid vibration, moving backwards and forwards several times in a second, and the discharges keep time with its movements.

541. Difference between Effects at Making and Breaking.—When the secondary circuit is closed, the quantity of electricity which traverses it in one direction in breaking the primary circuit is equal to that which traverses it in the opposite direction in making; but there is a great difference between the two currents. The current at breaking is much more sudden and violent than the other, standing to it in much the same relation that a blow stands to a steady pressure.

A similar difference between making and breaking is observed when there is only a single circuit. Whenever the two wires attached to the two poles of an ordinary battery are separated, a spark passes between them, but no spark passes in bringing them together.

542. Direction of Discharge.—When the terminals of the secondary coil are separated, leaving a proper interval of air between, discharge takes place across this interval at the instant of breaking the primary circuit, but not at the instant of making. The currents which pass are thus all in one direction. This direction is the same as that of the primary current. It is a general rule (whether soft iron be present to assist the action or not) that the stopping or weakening of the current in a wire induces (or tends to induce) a current in the same direction in any neighbouring wire, and that the starting or strengthening of a current induces an opposite current.

543. Magneto-electric Machines.—In magneto-electric machines, an electro-magnet of compact form called the *armature* is caused to rotate near the poles of a powerful fixed magnet, in such a manner that the core of the armature becomes magnetized first in one direction and then in the opposite, by the inductive action of the poles of the fixed magnet. Every change in the magne-

tization of the core induces a current in the coil wound upon it. Hence currents in alternately opposite directions are excited in this coil, their strength increasing with the speed of rotation.

The size of wire to be used for the armature varies according to the purpose for which it is intended. Thin wire allows a greater number of convolutions to be employed, and thus gives greater electromotive force. It is suitable for giving shocks and for sending currents through circuits of high resistance. Thick wire offers less resistance and gives a stronger current than thin, if the external resistance be small, It is suitable for such purposes as raising wires to incandescence. A coil of thick wire is usually called a *quantity* armature, and a coil of thin wire an *intensity* armature.

544. It is now usual in powerful machines of this class to employ electro-magnets as the fixed magnets, and the current which feeds these fixed magnets (called the *field-magnets*) is often the current generated by the machine itself. The machines are in this case called *dynamo* machines. When a dynamo machine has been standing idle, it gives hardly any current at starting, because there is very little magnetism remaining in the cores of the field magnets; but this magnetism is rapidly multiplied by the action of the machine, and full power is attained in perhaps less than a minute.

The name *dynamo*, or more fully *dynamo-electric* machine, was originally confined to machines which thus supply the current for their own field magnets; but it is now usually applied to any machines in which the field magnets are electro-magnets. Such machines, driven by steam-engines or other powerful motors, are now almost universally employed when electric currents are required on a large scale; for example, in electric lighting.

545. **Direct and Alternating Currents.**—The currents generated in the armature of any magneto or dynamo machine are alternately in one direction and the opposite; and in one class of machine the two ends

of the armature-coil are in permanent connection (by rubbing contacts on two cylinders) with the two ends of the external circuit, which accordingly receives *alternating currents*. In another class of machine the rubbing contacts are discontinuous, and are so arranged that one end of the external circuit receives only positive and the other end only negative electricity.

Machines of this latter class are called *direct-current* or *continuous-current* machines, and they can not only be used for generating currents when they are driven by mechanical work, but can also be driven by currents and will then furnish mechanical work. When thus used they are called *electro-motors*.

546. **Transmission of Power.**—Mechanical power can thus be transmitted to a distance by means of two successive transformations. A dynamo at the sending station transforms mechanical work into electric currents, and an electromotor at the receiving station retransmutes these currents into mechanical work. There is some waste in the process of transmission, but in many cases the waste is less than it would be by any other mode of transmission.

547. **Electro-medical Machines.**—Small magneto-electric machines, with intensity armatures, are very much used for electro-medical purposes. The patient holds two metallic handles connected with the ends of the coil, and receives a rapid succession of shocks when the machine is turned.

548. Another kind of electro-medical machine frequently used, is one in which, when the patient holds the handles, a current from a galvanic battery of moderate power passes through his body, and also through a long coil of wire which forms the remainder of the circuit. The patient feels nothing as long as the current is allowed to flow steadily; but a notched wheel is provided, by turning which the circuit is alternately made and broken; and every break gives the patient a shock.

549. **Induction of a Current on Itself.**—This

is an instance of what is called *the induction of a current upon itself.* It is more marked with a long wire than with a short one, more marked with a wire coiled up than with the same wire stretched out; and still more marked when the coiled wire surrounds a soft-iron core. We have seen, in art. 542, that the cessation of a current tends to produce an instantaneous current in the same direction in a neighbouring wire, especially if the latter be wound upon the same core. This remains true even when the two wires are merely different convolutions of one and the same wire; and this mutual induction of different parts of a coil gives rise to an instantaneous electromotive force, which is much more intense than that by which the steady current was maintained.

ELECTRIC LIGHT AND SPARK.

550. **Arc Light.**—The oldest and commonest form of electric light is called the *arc* light. It is obtained between two rods of carbon, which are first allowed to touch, so as to complete the circuit, and are then removed a small distance asunder. The current is not able to leap over the interval of air between the carbons, until contact has been made; but the spark which passes at breaking contact heats the intervening air; and as hot air conducts electricity, the current continues to pass until, from any cause, the interval become too great.

The intense light is due to the high temperature of the carbon terminals and of the small particles of carbon which fly across the interval.

The carbons are gradually consumed—the positive terminal about twice as fast as the negative, and special mechanism is provided for keeping the distance nearly constant. This mechanism is controlled by the strength of the current, according to various plans, all depending on the fact that increase of distance produces increased resistance and consequent weakening of the current.

551. **Incandescent Light.**—In the modern "incan-

descent" lamps the light is produced by sending the current through a filament of carbon inclosed in a very perfect vacuum. As there is no oxygen the carbon does not consume although it is white hot.

552. **Firing of Mines.**—The firing of mines by electricity is usually effected by means of a short piece of very fine platinum wire, which is heated to incandescence by a current, and ignites the charge.

553. **Nature of the Electric Spark.**—The electric spark consists of matter which has been rendered incandescent by the passage of electricity through it, just as wires are heated by the passage of a voltaic current.

When the spark is examined by the spectroscope, it shows bright lines, indicating incandescent vapour. These lines include both those due to the air or other gas through which the spark passes and those due to the vapour of the metallic terminals between which it passes. It is therefore to be inferred that small portions of the terminals are vaporized by the discharge.

554. The actual passage of the spark is doubtless preceded by an inductive action—some kind of tension or stress—which, when it attains a sufficient intensity, causes the resistance of the air to give way and allow the spark to pass. This action is most intense along the path of least resistance, which is accordingly the place where the rupture actually occurs. The crookedness which is observable in the spark when long, indicates the presence of inequalities (perhaps particles of dust), which render the path of least resistance crooked. The time occupied by the passage of a spark is excessively brief,—more instantaneous, perhaps, than any other phenomenon with which we are acquainted. According to a determination made by Wheatstone with the aid of a rapidly revolving mirror, it lasts for only the $\frac{1}{24000}$ part of a second. It therefore cannot be seen either to travel from one end towards the other, or to start from both ends and meet in the middle, but is visible along its whole length at once.

555. **Geissler's Tubes.**—Discharge occurs with

Fig. 155.

greater facility through rare than dense air. Glass tubes from 10 to 20 inches long (called Geissler's tubes), containing air or other gases in a state of extreme tenuity, and having platinum electrodes sealed into their ends so as to afford conducting communication between their interior and their exterior, readily allow discharge to pass through their interior from one of these electrodes to the other. The experiment is usually shown by connecting the two electrodes to the terminals of a Ruhmkorff's coil or of a Holtz's machine. Even the smallest induction coil is sufficient for the purpose.

The discharge in these tubes has not the appearance of a narrow line of light (like a spark in ordinary air), but of a broad nebulous mass, or series of masses, of faint luminosity, usually requiring a darkened room to render them visible. Fig. 155 gives a good idea of the appearance presented. A is the positive and B the negative terminal, the difference of appearance between the two ends being very marked. The alternate bands of light and darkness (constituting what is called the stratification of the discharge) are larger in proportion as the gas in the tube is more highly rarefied. If one part of the tube, as at C, is narrower than the rest, the light is most intense in this part.

556. Density of Minimum Resistance.—The resistance of a gas does not, however, decrease indefinitely with its density. It is possible to carry exhaustion beyond the point of minimum resistance, and even to go so far beyond it as to make the resistance much greater than that of air at

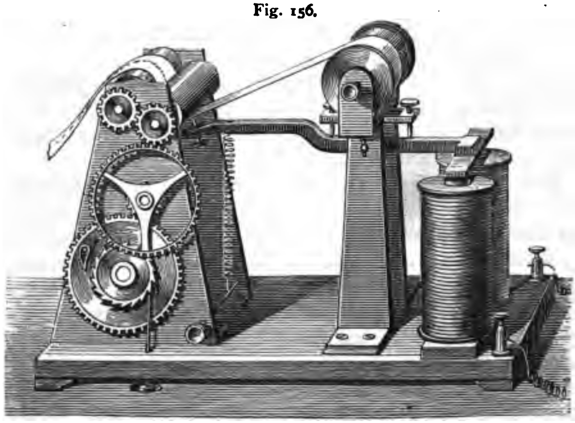
atmospheric density; but to obtain this result, very special means must be employed for perfecting the vacuum.

ELECTRIC TELEGRAPH.

557. There are various methods both of sending and of receiving telegraphic signals. On all public telegraphs the current is supplied by a battery; but many private telegraphs are worked by a small magneto-electric apparatus.

558. **Morse's Recorder.**—The receiving apparatus generally employed on public telegraphs is the electro-magnetic arrangement of Morse, represented, in its simplest form, in Fig. 156. The signals sent through the

Fig. 156.



line cause an electro-magnet, shown in the figure, to be alternately made and unmade, thus producing an up-and-down motion of an armature,[1] which forms one end of a

[1] The armature of an electro-magnet is a piece of soft iron whose function is to be alternately attracted by the electro-magnet and drawn away from it. It must not be confounded with the armature of a magneto-electric machine, which is something entirely different.

lever, the other end carrying the pen, which sometimes, as in the figure, is a mere point for scratching. The paper, in the form of a very long strip, is drawn past the pen by clock-work, which can be started and stopped at pleasure. Straight marks, with spaces between them, are thus produced on the paper; the length of a mark being proportional to the time during which a current passes, and the length of a space to the time during which it is interrupted. Two lengths of mark are employed, called *dot* and *dash*, and three lengths of space. Each letter is represented by a combination of dots and dashes, separated by the smallest space. A single dot is E, a single dash T, and the whole alphabet is constructed on the plan of giving the shortest signs to the letters of most frequent occurrence. The different letters of a word are separated by the space of medium length, and the different words by the longest space.

559. **Needle Telegraphs.**—In needle telegraphs the signals are given by deflections of an upright needle, a motion of its upper end to the left being equivalent to *dot*, and a motion to the right to *dash*.

560. **Relay.**—In connection with the Morse telegraph, great use is made of *relays*. A relay is an arrangement for closing and opening the circuit of a local battery (that is, a battery near at hand) by means of currents received from a distance, and thus employing the local battery to do work which the currents from a distance have not sufficient strength or sharpness to perform directly. The relay contains an electro-magnet, through whose coil all currents from the line wire must pass, on their way to earth. The movements of its armature to and from the electro-magnet, close and open the circuit of the local battery. Thus, if the coil of the receiving instrument, Fig. 156, be included in the circuit of the local battery, it will be actuated by currents from this battery contemporaneous with those which arrive by the line wire.

561. **Function of the Earth in Telegraphy.**—In the early experimental stages of electric telegraphy,

a complete circuit of wire was employed to connect the poles of the battery which sent the current; but it was soon found that the return wire might be dispensed with, by making proper connections with the earth. One pole of the sending battery must be connected with the earth, and one terminal of the receiving instrument must also be connected with the earth. Electricity will then be drawn from the earth at one end, and given back to the earth at the other. The earth is to be regarded as a reservoir of immense capacity, ready to give or receive any amount of electricity that may be required, at either station, rather than as a conductor through which the current returns to the battery which sent it.

562. **Telephone.**—For short distances, the telegraph

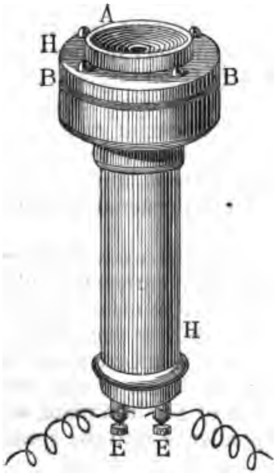


Fig. 157.—Telephone.

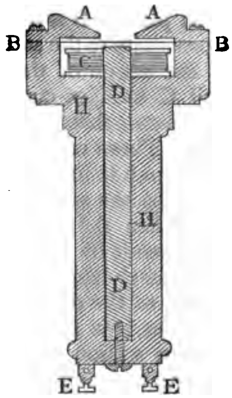


Fig. 158.—Section of Telephone.

has now been practically superseded by the telephone. Figs. 157, 158 represent the instrument which was originally employed both for transmitting and receiving, but

is now used for receiving only. D D is a steel magnet, C a coil of very fine silk-covered copper wire surrounding the magnet close to one end, and having its terminals in permanent connection with the two binding-screws E E. B B is a thin disc of soft iron held in its circumferential portion between the two parts of the wooden case H H, while its central portion is left free, and nearly touches the end of the magnet. A A is the mouth, which is held to the ear in receiving, and was spoken into when the instrument was used for sending. The two binding-screws E E are connected with two wires which lead to the other station.

563. When the instrument is used as a transmitter its action depends on the fact that a magnet, whether temporary or permanent, moved in the neighbourhood of any part of a closed circuit, induces a current in the circuit. The iron plate B B is a temporary magnet in consequence of its proximity to the steel magnet D, and when it vibrates under the action of the speaker's voice its influence on the coil C induces currents in the circuit to which this coil belongs—that is, the circuit connecting the two stations. These currents are of an undulatory character, and their undulations correspond to the aerial undulations produced by the speaker's voice.

564. When the instrument is used as a receiver its action depends on the principle that currents circulating round a steel magnet increase or diminish its strength according to the direction in which they circulate. Undulatory currents such as we have described arrive from the distant station, and every undulation of the current causes an attraction or repulsion of the iron plate, which accordingly vibrates in correspondence with the speaker's voice at the sending station.

565. The method now used for producing the undulatory currents depends on the variation of resistance at the point of contact of two conductors according as they more or less strongly pressed together. Carbon is ecially sensitive to such variations, and the contact

pieces are always either both carbon or one carbon and the other metal. The contact pieces are carried by a thin iron diaphragm which vibrates under the influence of the speaker's voice, and serve—something like the make-and-break spring in a Ruhmkorff's coil—to alter the strength of the primary current given by a small local battery. There are two small coils, like the primary and secondary coils in a Ruhmkorff's apparatus, and the secondary coil is in circuit with the wires leading to the further station. The currents which reach the receiving instrument are thus the secondary currents produced by the variations of strength of the primary current. It is to be understood that the primary current is not completely stopped and recommenced as in a Ruhmkorff, but only undergoes variations of strength. Much louder sounds can be produced in the receiving instrument by actually making and breaking contact, and this is done in the preliminary signal for calling attention.

EXAMPLES

On Resistance and Ohm's Law. Arts. 499–505.

1. One wire is 10 ft. long and ·02 inch in diameter, another is 50 ft. long and ·03 inch in diameter. Compare their resistances, their material being the same.

2. Compare the resistance of a copper wire 100 ft. long and ·025 inch in diameter with that of an iron wire 120 ft. long and ·015 inch in diameter, assuming the specific resistance of iron to be six times that of copper.

3. Two points are connected by two wires in parallel circuit. The resistance of one wire is 20 ohms and that of the other 30 ohms. Calculate the resultant resistance between the two points.

4. A current from a battery is flowing through an external circuit having a resistance 5 times that of the battery. In what ratio will the current change when we substitute an external resistance $2\frac{1}{2}$ times that of the battery?

5. Calculate the resistance of a battery from knowing that the current which it gives through a wire of 10 ohms resistance is double of that which it gives through a wire of 25 ohms resistance.

6. Compare the current given by a battery of 5 cells, each having a resistance of 1 ohm, through an external resistance of 15 ohms, with that given by a battery of 10 of the same cells through an external resistance of 20 ohms.

7. How many cells, each having a resistance of 2 ohms, will give the same current through an external resistance of 7 ohms that 30 give through an external resistance of 35 ohms?

ANSWERS.

Ex. 1. As 9 to 20. Ex. 2. As 1 to 20. Ex. 3. 12 ohms. Ex. 4. As 7 to 12. Ex. 5. 5 ohms. Ex. 6. As 3 to 4. Ex. 7. 6 cells.

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